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Multi-objective transmission expansion planning considering multiple generation scenarios

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ABSTRACT

This paper shows a methodology for solving the Transmission Expansion Planning (TEP) problem when Multiple Generation Scenarios (MGS) are considered. MGS are a result of the multiple load flow patterns caused by realistic operation of the network, such as market rules, availability of generators, weather conditions or fuel prices. The solution to this problem is carried out by using multiobjective evolutionary strategies for the optimization process, implementing a new hybrid modified NSGA-II/Chu–Beasley algorithm and taking into account variable demand and generation. The proposed methodology is validated using the 6-bus Garver system and the IEEE-24 bus system. The TEP is based on the DC model of the network and non-linear interior point method is used to initialize the population.

A set of Pareto optimal expansion plans with different levels of cost and load shedding is found for each system, showing the robustness of the proposed approach.

1. Introduction

The TEP consists on determining the required investment plan to reinforce the transmission network, in order to achieve minimum cost without load shedding. For finding an adequate plan, different aspects should be taken into consideration with the purpose of facing the new challenges that have arisen in the previous years.

Restructuring process in the electricity sector has led to a stronger interaction of technical and market aspects. Theoretically, these changes allow competition, promote higher quality and lead to better prices of the service. Planning and expansion in competitive markets should be characterized by low costs, quality, reliability and security, and accompanied by remuneration to equipment owners.

Planning also promotes network access for generators, as well as customers. The bridge to allow this access is the transmission network and all associated infrastructure, and consequently is the base for the electric market. In the case of the generation, the transmission network permits different dispatch scenarios and allows competition among the agents.

Under the previous premises, it becomes necessary to build a transmission network capable of taking advantage of future generation, supplying forecasted load, and avoiding potential congestion costs, which are at the end transferred to customers. The planning process and models must take into account investment and congestion costs, by analyzing possible dispatching scenarios resulting from market rules. The resulting power flow patterns become a test for planners, in order to model and find a suitable transmission system with plenty of capacity, and guaranteeing social welfare.

The mathematical model for planning the transmission system considers current system topology, the forecast of generation and demand, power balance equations, among others, and results in linear and non-linear algebraic expressions containing real and integer variables. Given the nature of model, it is considered as a Mixed-Integer Non-linear Programming (MINLP) problem.

1.1. Modeling and solving the TEP

The problem can be solved using a static approach [1–3] or a multistage model [4–8]. The Static approach considers only one generation-demand scenario, and the multistage or dynamic model takes into account several generation-demand periods of time.

Different mathematical representations have been proposed to solve the TEP. The main implemented models in order of complexity, are: transportation [9], hybrid [10], DC [4,11] and AC [12,13].

For solving the previous the mathematical models, different techniques and methods of solution have been used, such as linear

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List of symbols

c_{ij}	cost of circuit between buses $i-j$
f_{ij}	power flow between buses $i-j$
γ_{ij}	susceptance between buses $i-j$
n_{ij}	number of added circuits between buses $i-j$
n_{ij}^0	number of circuits in the base case between buses $i-j$
$\bar{f}_{ij}$	maximum power flow between buses $i-j$
$\bar{n}_{ij}$	maximum number of circuits between buses $i-j$
S	branch-node incidence matrix
g	generation vector
d	demand vector
w, w^k	fictitious generation vector for the base case and for the k-th generation scenario
f	vector of power flows
θ_i	voltage angle at bus i
Ω	set of candidate branches
Ω_1	subset of generators in the lower limit
Ω_2	subset of generators in the upper limit
N_b	number of buses
α	penalization factor of load shedding
δ	savings of supplying additional demand
d_{total}	total system demand
ng	number of generators
gs	number of generation scenarios
x^k	variable x evaluated in the k-th generation scenario
L_{max}	maximum allowed load shedding value
BPC	basic planning constraints
MPC	multiple generation scenarios constraints
P_t	parent population of the optimization algorithm in the generational cycle t
Q_t	offspring population of the optimization algorithm in the generational cycle t
v_m^{max}, v_m^{min}	maximum and minimum value of the objective function m
$v_m^{(I_{j+1}^m)}, v_m^{(I_{j-1}^m)}$	neighbor solutions for configuration j
r_i	rank of solution i
ρ_{div}	number of different bits for diversity check
ρ_{mut}	number of bits for mutation

programming [9,14,15], dynamic programming [16], non-linear programming [17], mixed-integer programming [18], Benders [19,20], and also decomposition techniques such as Branch-and-Bound [21]. Besides classical techniques, metaheuristic methods have also been satisfactorily used as an alternative, for instance, references [1,22–28] show how TEP is solved using Simulated Annealing, Tabu Search, Genetic Algorithms and Particle Swarm Optimization. Other recent metaheuristic optimization techniques, such as frog leaping, immune systems, ant colony, chaos and bee colony algorithms, have also been used as referenced in [3].

1.2. Planning the transmission network in a market environment

Deregulation in electricity markets have led to new challenges in the planning process. Under a market environment, the network expansion must ensure equity in access for all system participants, which leads to additional complications in the model. The following paragraphs summarize some of the proposed approaches to face these new challenges.

Reference [29] develops a multi-period model which takes into account nodal prices, line congestion, financial investment parameters and their relation with the amortization during the planning period. The model is validated on the Spanish network and different scenarios of demand and contingencies are used.

A multiobjective methodology is presented in [30], incorporating investment cost, congestion cost and reliability level, which are to be minimized. A multi-period model is solved and the NSGA-II algorithm is used to return a set of non-dominated solutions.

Author in [31] used an improved differential evolution model to address the TEP. Market considerations were included by calculating annual generation cost for different technologies and by adding annuitized cost of transmission. The author also performs a comparison of the results of differential evolution and the genetic algorithm for the IEEE 30-bus system.

The work presented in [32] solves the TEP by including the curtailment cost for bilateral transactions and the one associated to customers for spot market, besides the investment for new transmission equipment. This way, the network is reinforced in such a way that congestion constraints are alleviated in order to allow market transactions. Benders decomposition is used and the methodology is validated on the South-Brazilian system.

Particle Swarm Optimization was used in [33] to obtain optimal plans for Garver and IEEE 24-bus systems. The problem considers a multi-year model and the expansion process depends on the investment costs, and the social welfare. A performance comparison of different swarm approaches is also carried out.

Another market approach is shown in [34]. The objective function includes operation cost, load curtailment and investment cost for different load levels, with the idea of providing equity to all market participants. The model considers multiple stages and the solution is found by a genetic algorithm for Garver and IEEE 24-bus system.

The approach presented in [35] used a congestion surplus index through lagrange multipliers, in addition to the transmission investment and the expected energy not supplied. All of the objectives are to be minimized and the Strength Pareto Evolutionary Algorithm (SPEA) was used to obtain a set of Pareto optimal solutions.

An alternative for treating the described problem is shown in [36]. In this work, a procedure for network reinforcement in a deregulated environment is designed, different patterns for power flow are considered and a decision scheme is incorporated to minimize the risk of the selected plan. The authors design and select a number of generation scenarios with a probability of occurrence for a future year. This problem was also faced in [37] considering network security (N-1 contingency criteria). The way of solving this problem using a mono-objective approach is shown in [38].

1.3. About the present work

This paper proposes an approach for the TEP when full open access for generators is considered. As a result, multiple power flow patterns need to be analyzed in order to obtain a set of investment proposals. An enhanced multiobjective algorithm is used to obtain a set of Pareto optimal expansion plans with different level of investment and future load shedding. The solutions provide adequate operative conditions for any load flow pattern resulting from any dispatch scenario, ensuring low potential values of load shedding. This is achieved by considering the feasible Multiple Generation Scenarios (MGS) and also taking into account demand and generation as a variable in a narrow range. The proposed method

is validated using the Garver system and the IEEE 24-bus RTS test system.

The main contributions of this paper are listed below:

- Multiple load flow patterns are included in the model to reflect a more realistic planning process, by means of multiple generation scenarios and a multiobjective approach.
- Instead of one expansion proposal, several Pareto optimal expansion plans are obtained for both test systems. This methodology differs from most traditional planning schemes.
- An original multiobjective algorithm is presented as a tool for the community related to the electrical power systems and operational research fields. Performance analysis are carried out to demonstrate its convenience.
- Variable generation and demand in each bus is considered in the model and comparative analysis with fixed values is carried out to show the impacts on the investment costs.

1.4. Organization of the paper

The present work is organized as follows: Section 2 presents the mathematical formulation considering MGS. Section 3 shows the proposed multiobjective algorithm used for solving the TEP. Next, simulation tests and results are detailed in Section 4 for Garver and IEEE 24-bus system. Finally, conclusions are drawn in Section 5.

2. Mathematical model

When DC load flow model is used for representing the transmission network, the mathematical formulation of the static TEP considering load shedding is the following [39]:

$$\min \quad v = \sum_{ij\in\Omega} c_{ij} n_{ij} + \alpha \sum_{i\in N_b} w_i \tag{1}$$

$$\text{s.t.} \quad Sf + g + w = d \tag{2}$$

$$f_{ij} - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i - \theta_j) = 0 \tag{3}$$

$$|f_{ij}| \leqslant (n_{ij}^0 + n_{ij})\bar{f}_{ij} \tag{4}$$

$$0 \leqslant g \leqslant \bar{g} \tag{5}$$

$$0 \leqslant w \leqslant d \tag{6}$$

$$0 \leqslant n_{ij} \leqslant \bar{n}_{ij} \tag{7}$$

$$ij \in \Omega, n_{ij} \text{ integer} \tag{8}$$

Eqs. (2) and (3) represent the first and second Kirchhoff laws, respectively.

This problem is usually divided into two sub-problems. The first, is the investment problem and has the objective of determining the expansion plans that should be evaluated. The investment plans might have certain level of infeasibility, which is evaluated by the second subproblem: the operative [2,24,26,37]. The latter results in a Linear Programming (LP) problem when the investment proposal is known, which is solved in this paper using an Interior Point Method. For more information regarding the implementation of this method, readers are advised to examinate reference [40].

These two problems are iteratively solved until a feasible minimum cost plan is found.

2.1. TEP considering multiple generation scenarios

In order to take advantage of future generation and to supply necessary power for future loads, the transmission network must be reinforced based on the existence of deregulated markets. To face this new scheme, different load flow patterns must be taken into account according to the dispatch scenarios created by the market rules, by the changes in generation and demand, and by the availability of primary energy sources [41–43].

When considering the multiple load flow patterns, the different combinations of generated power in the electric system should be considered. These combinations depend on the cost of the MWh of each plant, weather conditions, the hourly demand, bids, and in general, the market rules of the specific location. Network should be able to deliver power without load shedding, hence, operative conditions must be evaluated for all possible generation scenarios, which is the purpose of the presented methodology. Along with the MGS, planners in a specific country should consider detailed market rules.

For a system with demand (d) and generation (g) level, a generation scenario is defined as any dispatch (within the generation limits) capable of meeting total demand, as follows:

$$\sum_{i=1}^{ng} g_i = d_{total} \tag{9}$$

Given the real nature of the active power, infinite feasible scenarios can be found. The concept of feasible extreme scenario is used to generate a representative set of scenarios, which consists on setting the generators at the upper or lower limit. However, constraint (9) might not be met for a large number of combinations. To face this problem, practical extreme scenarios are considered, by dispatching $ng - 1$ generators in their upper/lower limits while the remaining generator dispatches the necessary power to establish generation-demand balance according to (9) [38].

According to the previous ideas, a feasible extreme scenario must comply with the following constraints:

$$\sum_{i\in\Omega_1} g_i + \sum_{i\in\Omega_2} g_i \leqslant d_{total} \tag{10}$$

$$\sum_{i\in\Omega_1} g_i + \sum_{i\in\Omega_2} g_i + g_q^{max} \geqslant d_{total} \tag{11}$$

It is important to point that the number of possible scenarios is large, and can be calculated as $ng \times 2^{ng-1}$. Depending on knowledge of the specific test system and the market, the number of scenarios can be reduced to avoid prohibitive computational time, however, the present work considers all feasible scenarios to test the proposed algorithm and methodology under the most extreme conditions.

2.2. Mathematical formulation with MGS

The formulation of the TEP when market is considered through MGS, and using the DC model, is the following:

$$\min \quad v = \sum_{ij\in\Omega} c_{ij} n_{ij} + \alpha \sum_{k=1}^{gs} \sum_{i\in N_b} w_i^k \tag{12}$$

$$\text{s.t.} \quad Sf^k + g^k + w^k = d \tag{13}$$

$$f_{ij}^k - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i^k - \theta_j^k) = 0 \tag{14}$$

$$\left|f_{ij}^k\right| \leqslant (n_{ij}^0 + n_{ij})\bar{f}_{ij} \tag{15}$$

$$0 \leqslant g^k \leqslant \bar{g}^k \tag{16}$$

$$0 \leqslant w^k \leqslant d \tag{17}$$

$$0 \leqslant n_{ij} \leqslant \bar{n}_{ij} \tag{18}$$

$$ij \in \Omega \tag{19}$$

$$k = 1, 2, \ldots, gs \tag{20}$$

$$n_{ij} \text{ integer} \tag{21}$$

The objective with this modeling is to obtain an expansion plan that meets the demand under any generation scenario. It is worth to notice that this mathematical model adds even more complex-

ity, given that the number of variables increases when MGS are included. This problem can be solved under a mono-objective or multiobjective approach, given that load shedding and investment are conflicting objectives.

2.3. Considering variable demand

Traditional planning schemes consider fixed demand as a result of load forecasting for a given horizon. It is possible to include demand in the problem as a variable due to load forecasting uncertainty. In this work, load in all buses is considered as a variable, allowing the demand to vary within certain range. The objective function for the operative subproblem has to be adjusted including demand:

$$\min\ v = \alpha \sum_{k=1}^{gs} \sum_{i \in N_b} w_i^k - \delta \sum_{i \in N_b} d_i \tag{22}$$

It is considered that $\alpha > \delta$, indicating that load shedding is more severely penalized than supplied demand. These two factors impact the objective function as a linear combination of load shedding and demand, so any combination that meets $\alpha > \delta$ implies that more importance is given to alleviating 1 MW of load shedding than supplying one additional megawatt of load. The minus sign in (22) shows that when the investment plan allows meeting a larger demand, the objective function tends to reduce its value, and the plan becomes attractive.

A new constraint must be also added to the model to consider demand variations:

$$d_{min} \leqslant d \leqslant d_{max} \tag{23}$$

In general, a narrow range for variations of demand should be enough, since load forecasting studies in power systems usually provide high, medium and low demand scenarios.

2.4. Multiobjective formulation

Investment in the TEP is conflicting with load shedding levels. In the proposed multiobjective formulation, low load shedding values are accepted to form a Pareto optimal set of expansion plans with different cost levels. This allows multiple choices for decision makers regarding the selection of one plan, according to higher level information.

The complete mathematical formulation is expressed by:

$$\min \quad \{v_1, v_2\} \tag{24}$$

$$\text{s.t.} \quad v_1 = \sum_{ij \in \Omega} c_{ij} n_{ij} + \alpha \sum_{i \in N_b} w_i \tag{25}$$

$$v_2 = max\left\{\sum_{i \in N_b} w_i^k\right\} \tag{26}$$

$$\sum_{i \in N_b} w_i^k < L_{max} \tag{27}$$

$$n = n_{ij} \in BPC \tag{28}$$

$$n = n_{ij} \in MPC \tag{29}$$

$$k = 1, 2, \ldots, gs \tag{30}$$

L_{max} is the maximum allowed load shedding, which depends on the total demand of the specific system.

Eq. (25) calculates the first objective function to be minimized, which is the cost of the expansion plan, and penalized if load shedding in the base case (without MGS) is different from zero. It is important to clarify that when certain expansion plan n_{ij} is analyzed, objective function (26) measures the most critical out of the k generation scenarios. This is, objective function v_2 is the highest load shedding in MW, which is obtained after calculating and comparing the operative conditions for each scenario. From the previous ideas, it can be inferred that for each investment proposal (n_{ij}), k operative subproblems need to be run in order to obtain the corresponding load shedding levels, although only the maximum is selected at the end.

For tests with variable demand, constraint (23) should be added.

3. Solution of the multiobjective formulation

When planning the transmission system to eliminate congestion under any generation scenario, the associated cost increases. Under a single-objective formulation, to achieve zero load shedding, expansion plans present high costs. Hence, it becomes important to explore other expansion plans with lower cost, allowing certain levels of non-supplied load. The present approach proposes a multiobjective planning scheme that allows low levels of infeasibility, idea expressed in Eq. (26). This multiobjective proposal presents a first objective to be minimized, which is the cost of the expansion plan, and the second objective measures the load shedding of the most critical scenario. These two objectives are expressed respectively in Eqs. (26) and (25).

It is clear that these two objectives are conflicting, given that low investment in the transmission system, tends to generate considerable load shedding, and vice versa. This characteristic justifies the importance of using a multiobjective approach.

To implement the multiobjective algorithm, investment proposals (n_{ij}) are evaluated using formulation (24)–(30), to obtain the values of both objectives: v_1 and v_2. In order to obtain a set of solutions with minimum levels of cost and load shedding, a multiobjective algorithm has to be implemented, as explained in the next subsections.

3.1. Concept of dominance

Most of the multiobjective algorithms use the concept of dominance, which consists on comparing two solutions to determine which one dominates the other. In the case of this work, the objective functions must be minimized, so it is said that solution x^1 **dominates** x^2 if these conditions are met [44]:

- $v_m(x^1) \leqslant v_m(x^2)$ for $m = 1, 2, \ldots, M$.
- $v_m(x^1) \leqslant v_m(x^2)$ for at least one $m\prime \in m = 1, 2, \ldots, M$.

When the first condition is not met by any of the two solutions, it cannot be stated which of them dominates the other. When this happens the solutions are **non-dominated**.

This concept can also be extended to find a set of non-dominated solutions belonging to a population. Reference [44] shows in detail the procedure for finding the set of non-dominated solutions when N individuals and M objective functions are considered.

3.2. Pareto optimality and ranking of solutions

When treating a multiple objective problem, the concept of optimal solution changes. For a multiobjective problem presenting conflicting objectives a set of trade-off solutions should be found, and that set must be formed by non-dominated solutions taken from the analyzed population.

When any individual from the set of non-dominated solutions dominates any other solution remaining in the population, the non-dominated set is called **Pareto Front**.

For a multiobjective problem, the optimal Pareto front should be found, that is, finding the best non-dominated set of solutions. When analyzing a set of solutions, a sorting is carried out in order to determine the number of Pareto fronts in a population, and to assign each solution an attribute called the ranking (r). This process

is done based on the value of the objective functions, which in the case of this work, result from solving the problem (24)–(30) to obtain v_1 and v_2.

The solutions in the best Pareto front in a population are assigned ranking r_1, and so on until the worst front. This attribute helps in determining the quality of a solution by its presence in a determined Pareto front, and is key to understand how the proposed genetic operators work, as shown in Sections 3.4.7 and 3.4.3.

3.3. Elitist non-dominated sorting genetic algorithm: the basic NSGA-II

This evolutionary algorithm was proposed in the year 2000 [45,46]. In the NSGA-II, the offspring set Q_t of size *NP*, is created from the parents population P_t also of size *NP*. The offspring population is created using tournament selection, crossover and mutation. After this process, both populations are merged together to create a new population R_t of size 2*NP*. Next, objective functions of R_t are evaluated and classified through a non-dominated sorting in different Pareto fronts. Once the sorting process is terminated, a new population is generated from the solutions in the best non-dominated fronts. This new population is created using the solutions in the best Pareto fronts until *NP* solutions conform the new set. When the size of the last Pareto front entering the new population exceeds the number of remaining slots, those with larger distance to their neighbors are selected in order to preserve diversity.

To obtain an idea of the density of solutions around a solution *i*, the average distance to two surrounding solutions is calculated, based on the values of the objective functions. This distance is used as an estimation of the perimeter of the cuboid, formed by using the closest neighbors as vertices, as shown in Fig. 1.

The crowding distance ($d_{I_j^m}$) calculation for each solution *j*, according to an index *I*, can be found using the following expression:

$$d_{I_j^m} = d_{I_j^m} + \frac{v_m^{(I_{j+1}^m)} - v_m^{(I_{j-1}^m)}}{v_m^{max} - v_m^{min}} \tag{31}$$

The distances consider all of the objective functions, and infinite value is assigned to the extreme solutions in the analyzed Pareto front, given that they have the best value in one of the objective functions.

Then, when each neighbor of the $j-th$ solution is taken into account, the objective functions are sorted in ascending or descending order so that each distance can be evaluated. The Crowding distance assignment algorithm is shown in Algorithm 1:

Algorithm 1. Crowding-sort $(F, <_c)$ [47]

$l \leftarrow |F|$
$d_i \leftarrow 0$ for each i in the set
for $m = 1 : M$ **do**
  $I^m \leftarrow$sort $(v_m, >)$
  $d_{I_1^m} \leftarrow \infty$
  $d_{I_l^m} \leftarrow \infty$
  for $j = 2 : l - 1$ **do**
    $d_{I_j^m} \leftarrow$by using Eq. (31)
  end for
end for

The use of the distance of a solution is the key for preserving diversity in the NSGA-II, which is very important in population based algorithms. This methodology tends to privilege less surrounded solutions to promote them into the next generational cycles. In the case of TEP, this allows searching expansion plans in wider areas of the search space, and disregard investment proposals with similar values of objective functions.

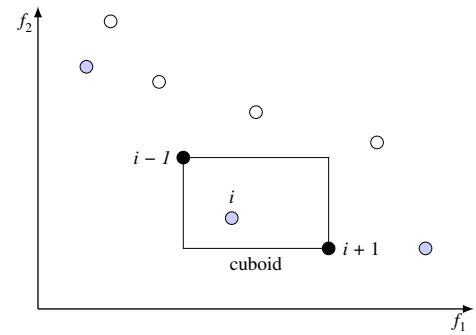

Fig. 1. Neighbor solutions for the *i*-th configuration.

After crowding sort of population R_t, those solutions within the first Pareto fronts are promoted to the next generational cycle, always based on both criteria: better range or larger distance. The complete NSGA-II algorithm is described in pseudocode Algorithm 2.

Algorithm 2. NSGA-II Algorithm [47]

Data$\leftarrow$ Branches, Buses, Demand, Generation
$P_0 \leftarrow$ Random
$F \leftarrow$ Non-Dominated sorting (P_0)
Distances $\leftarrow$ Crowding-sort $(F_i, <_c)$
$Q_0 \leftarrow$ Selection, Recombination, Mutation (P_0)
for $t = 1 : T$ **do**
  $R_t \leftarrow P_t \cup Q_t$
  $F \leftarrow$ Non-Dominated sorting (R_t)
  $P_{t+1} \leftarrow \emptyset$
  $i \leftarrow 1$
  while $|P_{t+1}| + |F_i| < N$ **do**
    $P_{t+1} \leftarrow P_{t+1} \cup F_i$
    $i \leftarrow i + 1$
  end while
  Distances $\leftarrow$ Crowding-sort $(F_i, <_c)$
  $P_{t+1} \leftarrow$ Include the first $(N - |P_{t+1}|)$ elements of F_i
  $Q_{t+1} \leftarrow$ Selection, Recombination, Mutation (P_{t+1})
  $t \leftarrow t + 1$
  end for

3.4. Enhanced multiobjective algorithm

This paper proposes a novel multiobjective algorithm based on the main advantages of the NSGA-II and the CBGA. The original NSGA-II creates an offspring (Q_t) population in each generational cycle; after this, it is combined with the Parents, obtaining the set of solutions R_t. This fact generates an important computational effort due to the need of calculating *NP* objective functions for the set Q_t in each cycle. For reducing computational effort and improving the performance of the multiobjective approach, some of the features of the CBGA are included. The CBGA creates only one offspring per cycle and maintains the population size constant, hence, reduces the number of times the objective function is calculated. The solution found in each cycle is included as a parent, based on Pareto-optimal theory as described in the next paragraphs.

3.4.1. Initialization

The process starts solving the non-linear problem of TEP, which is a relaxed version of problem (12)–(21) [40]. After this, real values for the additions n_{ij} are obtained, which might be important in the final solution to achieve feasibility. Given the structure of the continuous problem, these paths have low cost to power

transfer ratio, and are also relevant for alleviating load shedding problems. Although the continuous solution shows an interesting indication of important paths, it is not totally secure that all of the paths would be present in the final solution.

So said, this solution is used to generate only a few individuals, and the decision of adding a line where $n_{ij} \neq 0$ is taken randomly, and this way the individual has line additions in some of the paths meeting $n_{ij} \neq 0$.

After this step, the individual generation is carried out blocking the paths with $n_{ij} \neq 0$ and solving another non-linear problem. This leads to discovering other important paths that are not present in the base case and that have also certain importance in the planning process. The generation of the remaining individuals is then a cyclic process of blocking paths, running the non-linear problem and assigning additions, repeated a number of times depending on the population size.

3.4.2. Diversity verification

After the population is created, diversity check is carried out among the individuals, by comparing each one of the solutions, and ensuring that they are different in at least ρ_{div} bits.

The previous procedure ensures a controlled initialization to avoid large number of lines in the initial population and also spreads the solutions in the search space, which is even more important in multiobjective approaches and population based algorithms.

3.4.3. Selection

In the selection process two crowding distance tournaments are carried out in order to select two parents. Since each solution is evaluated using the formulation (24)–(30), then two important attributes can be calculated for each one of them by means of v_1 and v_2: ranking (r_i, presence in a specific Pareto front) and distance (d_i, measure of diversity). In each tournament kk parents are competing, and one of them is selected according to the crowding tournament selection operator, which is based on the rank r_i of the selected parents and the associated crowding distance d_i. The pseudocode of the proposed procedure is shown in Algorithm 3:

Algorithm 3. Crowding Tournament Selection [47]

```
for i = 1 : 2 do
  Q_1, Q_2, ..., Q_kk ← Random (P)
  j ← index (min(r_1, r_2, ..., r_kk))
  Q^best ← Q_j
  if | Q^best |= 1 then
    Parent_i ← Q^best
  else
    o ← index (min(d_1, d_2, ..., d_kk))
    Parent_kk ← Q_o
  end if
  i ← i + 1
end for
```

It is important to note for each tournament, that the solution with better rank is selected as a parent, or the less surrounded one (larger distance) when the rank of competing parents is the same. In conclusion, this operator tends to select the better ranked solutions from the Pareto optimality standpoint and the most diverse ones, which makes it an elitist operator.

3.4.4. Crossover

This work uses single point crossover for parents combination. It is important to point that in this enhanced approach, no crossover probability is predefined given that the population (P) remains the same and only one individual is allowed to enter the population if diversity and Pareto optimal conditions are met. After crossover of parents, two offspring solutions are generated and analyzed in order to keep only one, according to the rank and distance methodology followed in the selection stage. This is done by comparing both offspring with the entire population P_t and sorting this extended temporary set. After this, the offspring with the best features from the Pareto optimality and distance logic standpoint, is selected, and the other one is disregarded. This ensures that the offspring leads to interesting Pareto and distance based optimal regions.

3.4.5. Mutation

In this stage ρ_{mut} branches are randomly chosen in order to add or remove circuits. The decision of adding or removing a circuit is also based on a random parameter. In the case of this paper, a 50/50% probability was chosen.

3.4.6. Improvement

Another feature of the proposed algorithm is an improvement procedure consisting in analyzing the solution outcome from the mutation stage. This offspring is subject to a circuit redundancy analysis in order to determine if Pareto optimality can be improved, by temporarily retiring circuits and check if Q is still feasible. The drawback of this process is the increase of the computational effort, but the trade-off is the possibility of leading the algorithm towards high quality regions. The outline of this stage is described in Algorithm 4.

Algorithm 4. Improvement

```
if Q infeasible then
  Ordered ← sort circuit costs in descending order
  Q_original ← Q
  for j = 1 : Branches do
    Q(Ordered(j)) ← Q_original(Ordered(j)) − 1
    if Q infeasible then
      Q(Ordered(j)) ← Q_original(Ordered(j))
    end if
    j ← j + 1
  end for
end if
```

3.4.7. Promotion

To include an offspring Q into the population, a number of criteria must be met in order to ensure that good quality solutions are promoted to the next generational cycles. In this case, both Pareto optimal and diversity criteria are taken into account as shown in Algorithm 5.

Before this procedure is carried out, the population P_t and the offspring Q are merged together in order to perform the ranking of the complete population R_t. This is done by analyzing the information of all objective functions v_1 and v_2 of R_t, which is in turn obtained after solving (24)–(30). In general, this stage includes the offspring into the next generational cycle if it is diverse and belongs to a Pareto front that is best than the current worst, in the attempt of constantly improve the quality of the population. If the offspring is located in the first Pareto front (r_1) and it differs from all other solutions, it is also included. Is important to note that the only attribute that is taken into account in this stage is the raking, and not the distance. Diversity of the offspring is directly measured by the number of different bits when compared with the rest of the population, as explained in Section 3.4.2.

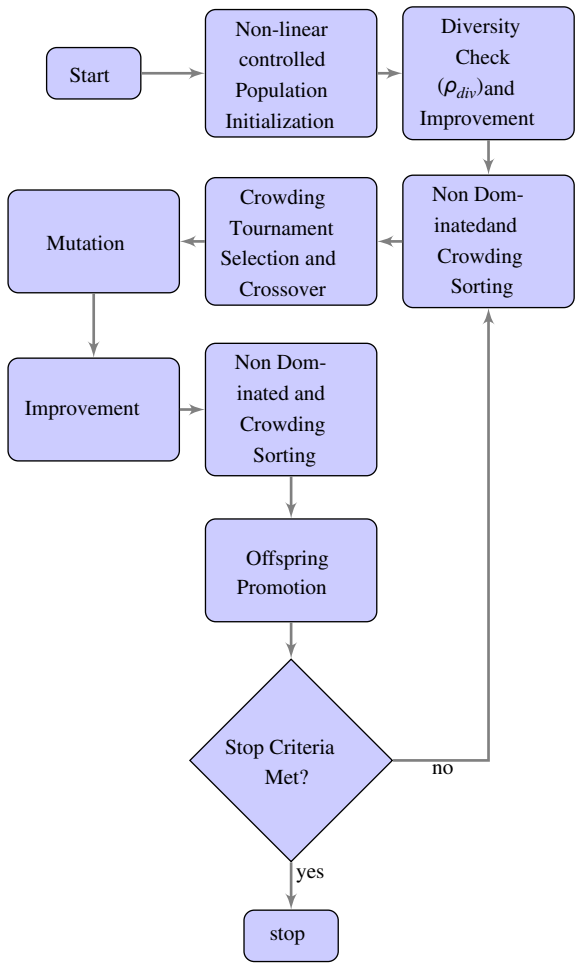

Fig. 2. Scheme of the enhanced NSGA-II.

For keeping the population size constant, when the candidate Q meets promotion criteria, the worst individual in the population is selected to be replaced. This selection is simple: The solution with the lower distance in the worst Pareto front.

Algorithm 5. Promotion

if $Q_t \in P_t$ **then**
  $P_{t+1} \leftarrow P_t$
else if $r_Q = 1$ **then**
  $P_{t+1} \leftarrow include(Q_t)$
else if $r_Q < rank^{max}$ and Q diverse **then**
  $P_{t+1} \leftarrow include(Q_t)$
else
  $P_{t+1} \leftarrow P_t$
end if

The structure of the complete proposed algorithm is shown in Fig. 2, and the stop criteria consists on a predetermined number of PLs without improving the best Pareto front, as detailed in the next section for each test system. Besides the decrease in the number of operative problems calculated in each iteration, the computational complexity is reduced to $O(M(1+N/2)^2)$, whereas in the original NSGA-II is $O(MN^2)$.

Table 1
Generation scenarios for Garver system.

Scenario	Generation (MW)		
	Bus 1	Bus 3	Bus 6
1	0	160	600
2	150	10	600
3	0	360	400
4	150	360	250

4. Tests and results

The problem formulated in (24)–(30), is solved using the enhanced multiobjective algorithm described in the previous section and programmed in Matlab R2011a environment. Two test systems from the specialized literature were used: the 6-bus system proposed by Garver [9], and the IEEE 24-bus system. Network data for these systems can be found in [19,10,36,48].

First, fixed demand model is investigated and Pareto fronts are shown for both systems, along with the obtained expansion plans. Then, a set of Pareto optimal expansion plans is shown for ±5% uncertainty in demand, demonstrating the increase in supplied demand and decrease in cost. L_{max} was set to 10% and 5% for Garver and IEEE 24-bus system, respectively. The algorithm was initialized with the scheme proposed in Section 3.4.1.

4.1. Garver 6-bus system with fixed demand

This network has 6 buses, 15 branches, a total demand of 760 MW, and a maximum of 5 parallel circuits to be added.

For this test system there are 4 feasible scenarios according to (10)-(11), which are detailed in Table 1. Scenario one, corresponds to generation in bus 1 at the minimum level, in bus 6 at the maximum, and generator in bus 3 is free to match the demand of 760 MW. Using the same logic, the other three generation scenarios are created with combinations of maximum, minimum levels, and one free generator.

As shown in the table, generation in buses 3 and 6 are important to create feasible scenarios. All of them have non-zero generation value in those specific buses. It is clear that each scenario equals the necessary demand, 760 MW.

The parameters used for this system were the following: 50 individual population, $\rho_{div} = 5$, $\rho_{mut} = 4$ and $kk = 2$, and stop criteria is set to 5000 PLs without improving the best Pareto front.

The enhanced algorithm found the Pareto front depicted in Fig. 3, and the corresponding circuit additions for the seven plans are shown in Table 2. The most critical scenario for each plan ranges from 70 to zero MW, and the cost varies from 200 to 268×10^3 USD respectively.

The extreme point with zero load shedding (268×10^3) is the one reported in [37] and improves the one reported in [49] by 2×10^3 USD, for single objective planning. This shows that the presented multiobjective approach contains that specific expansion plan and six additional options. The basic TEP solution is also obtained in the Pareto front, which corresponds to an investment of 200×10^3 USD.

4.2. Garver 6-bus system with variable demand

When 5% variation in demand and generation is considered and the same algorithm parameters are used, the obtained Pareto front is the one in Fig. 4 with the corresponding circuit additions in Table 3. It is concluded after comparing both cases, that relaxing generation and demand within certain limit, leads to a decrease in the cost of the expansion plans. For the zero load shedding case,

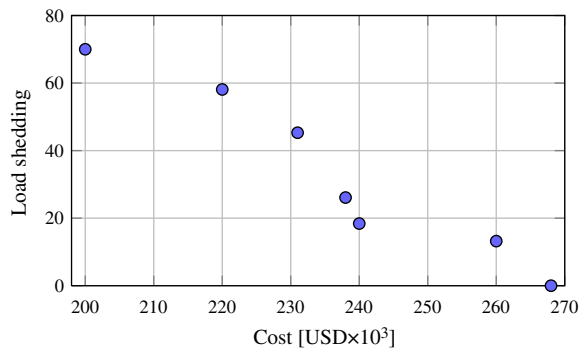

Fig. 3. Pareto front for Garver system without demand uncertainty.

Table 2
Expansion plans for Garver system with fixed demand.

Cost (10^3 U$)	r_g (MW)	Circuit additions
268	0.0	$n_{2-6}=4, n_{3-5}=2, n_{3-6}=1, n_{4-6}=2$
260	13.2	$n_{1-5}=4, n_{2-3}=2, n_{2-6}=1, n_{3-5}=2, n_{4-6}=2$
240	18.4	$n_{2-3}=1, n_{2-6}=4, n_{3-5}=2, n_{4-6}=2$
238	26.1	$n_{2-6}=3, n_{3-5}=2, n_{3-6}=1, n_{4-6}=2$
231	45.3	$n_{2-6}=3, n_{3-5}=1, n_{4-6}=2, n_{5-6}=1$
220	58.1	$n_{2-3}=1, n_{2-6}=4, n_{3-5}=1, n_{4-6}=2$
200	70.0	$n_{2-6}=4, n_{3-5}=1, n_{4-6}=2$

the cost is reduced 30×10^3 USD and the total supplied load increases to 798 MW.

4.3. IEEE 24-bus system with fixed demand

For this case the parameters were: population size of 100 individuals, $\rho_{mut}=4, kk=2, \rho_{div}=9$ and L_{max} set to 5% of the total demand, 427.5 MW, and stop criteria is set to 1 million PLs without improving the best Pareto front. This system has 178 feasible scenarios, which leads to higher computational effort. The best obtained Pareto front is shown in Fig. 5.

Besides The zero load shedding plan (1330×10^6 USD), which was also reported in [38], there are 38 additional expansion plans with different levels of load shedding and cost. Under the maximum load shedding permitted, the investment of the less expensive plan is 756×10^6 USD with a maximum load shedding of 418.99 MW. It is interesting to point that the zero load shedding plan has seven alternative optima which have not been previously reported and are shown in Table 4. In addition, the solution obtained in the present work for zero load shedding, improves the one reported in [49], which has a cost of 1477×10^6 USD. The solution in references [37,36] presents lower investment costs,

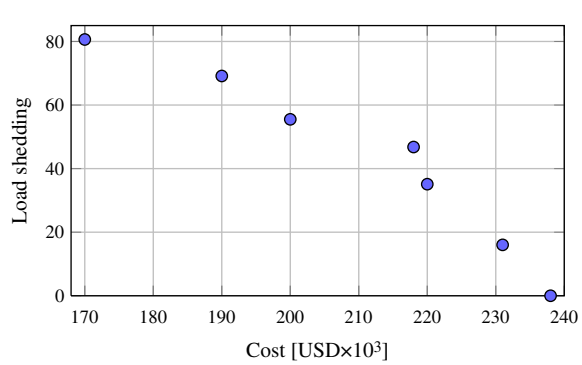

Fig. 4. Pareto front for Garver system with variable demand.

Table 3
Expansion plans for Garver system with variable demand.

Cost (10^3 U$)	r_g (MW)	Circuit additions
238	0.0	$n_{2-6}=3, n_{3-5}=2, n_{3-6}=1, n_{4-6}=2$
231	16.01	$n_{2-5}=1, n_{2-6}=4, n_{3-5}=1, n_{4-6}=2$
220	35.1	$n_{2-3}=1, n_{2-6}=4, n_{3-5}=1, n_{4-6}=2$
218	46.79	$n_{2-6}=3, n_{3-5}=1, n_{3-6}=1, n_{4-6}=2$
200	55.5	$n_{2-6}=4, n_{3-5}=1, n_{4-6}=2$
190	69.14	$n_{1-5}=1, n_{2-6}=3, n_{3-5}=1, n_{4-6}=2$
170	80.62	$n_{2-6}=3, n_{3-5}=1, n_{4-6}=2$

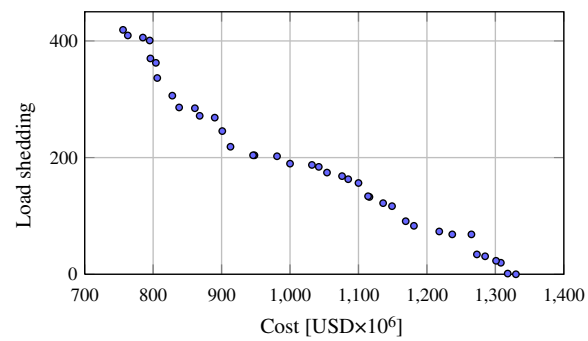

Fig. 5. Pareto front for IEEE 24-bus system with fixed demand.

Table 4
Alternative optima for the seven zero load shedding plans with Fixed demand. IEEE 24-bus system.

Branch	Circuit additions						
	1	2	3	4	5	6	7
01–02	1	1	1	1	1	1	1
01–05	1	1	1	1		1	
03–24	1	1	1	1	1	1	1
04–09	1	1	1	1	1	1	1
05–10	2	2	2	2	1	2	1
06–10	2	2	2	2	2	2	2
07–08	2	2	2	2	2	2	2
08–09	2	2	1	1	1	1	1
08–10	1	1	2	2	3	2	2
09–11			1				1
09–12	1	1		1	1	1	
10–11	2	1	1	1	2	2	1
10–12	1	2	2	2	1	1	2
11–13	1		1		2	1	1
11–14	1	1	1	1	1	1	1
12–13		1		1			
12–23	1	1	1	1	1	1	1
14–16	2	2	2	2	1	2	2
15–16							2
15–21	1	1	1	1	2	1	1
15–24	1	1	1	1	1	1	1
16–17	2	2	2	2	1	2	1
16–19	1	1	1	1		1	1
17–18	1	1	1	1	2	1	1
18–21					1		
20–23	1	1	1	1	1	1	1
21–22	1	1	1	1	1	1	1
01–08	1	1	1	1		1	1
02–08					1		1

given that those authors did not take into account the 178 scenarios but only 4, hence, the solution cannot be directly compared with the one obtained here for zero load shedding.

Boxplot in Fig. 6, shows a more graphical idea for load shedding distribution of each expansion plan. For all cases, minimum load shedding is zero. However, for configurations 2–9 there is a high number of scenarios with zero load shedding, given that most of data are outliers.

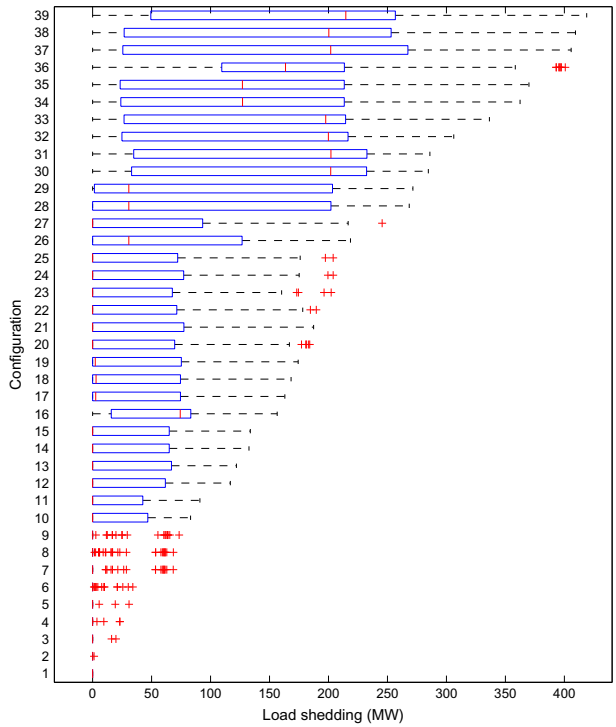

Fig. 6. Load shedding boxplot for IEEE 24-bus system with fixed demand.

In addition, configuration 16 has higher values of v_2^{mean} and infeasible scenarios, in spite of having less v_2^{max}. The selected metric only considers maximum values of load shedding, hence this plan results more attractive when compared to other configurations with less v_2^{mean} and infeasible scenarios, such as plans 17–25 or 27. The previous idea leads to the possibility of exploring different metrics for objective function 2, and reveal interesting information for some configurations.

Moreover, there are configurations with a median of zero (quartile Q2). This means that a high number of scenarios have zero load shedding (50% or more).

It is also concluded that configuration 15 has high load shedding values, distributed in quartiles Q1 and Q3. Therefore, there is a high density of data with non-zero load shedding and the median is also higher when compared to surrounding configurations.

On the other hand, when the impact of the 178 scenarios is analyzed, it can be concluded that the most critical scenarios are related to low generation levels in buses 1, 2, 7, 16 an 21. This information is relevant to discard scenarios in real life systems given their low probability of occurrence.

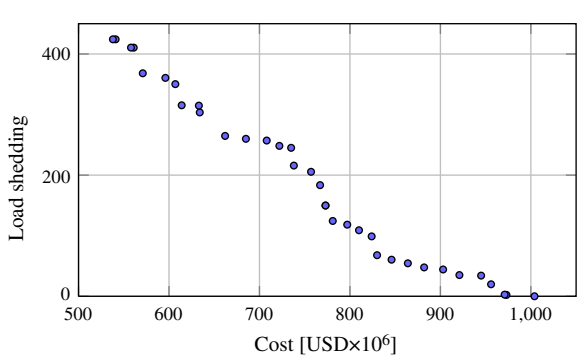

Fig. 7. Pareto front for IEEE 24-bus system with variable demand.

4.4. IEEE 24-bus system with variable demand

In this test, ±5% variation in demand at each bus is taken into account, and an upper bound of $1.05\bar{g}$ for generation.

The parameters used were the following: 100 individual population, $kk = 2, \rho_{mut} = 4, \rho_{div} = 10$ and stop criteria is set to 1 million PLs without improving the best Pareto front.

One of the extreme points of the Pareto front in Fig. 7, has a cost of 1004×10^6 USD and zero load shedding, with the following additions:

$$
\begin{aligned}
&n_{01-02} = 1, \quad n_{01-05} = 1, \quad n_{03-09} = 1\\
&n_{03-24} = 1, \quad n_{04-09} = 1, \quad n_{05-10} = 1\\
&n_{06-10} = 2, \quad n_{07-08} = 3, \quad n_{08-09} = 2\\
&n_{08-10} = 1, \quad n_{09-11} = 1, \quad n_{09-12} = 1\\
&n_{10-11} = 1, \quad n_{10-12} = 1, \quad n_{11-14} = 2\\
&n_{12-13} = 1, \quad n_{14-16} = 1, \quad n_{15-24} = 1\\
&n_{16-17} = 1, \quad n_{16-19} = 1, \quad n_{01-08} = 1\\
&n_{14-23} = 1
\end{aligned}
$$

Besides this expansion plan, 33 additional configurations are found.

When the Pareto fronts for fixed demand and uncertainty are compared, it is concluded that the latter leads to an important decrease in investment.

Boxplot in Fig. 8 depicts load shedding for each configuration according to each generation scenario. In all cases, minimum load shedding is zero. For solutions 2–15, there is large number of scenarios with zero load shedding, given that all data are outliers.

For solutions 16–19 the median is zero, which means that a large number of generation scenarios are feasible. Again, there are interesting expansion plans that lead to cost reduction in spite of having certain level of infeasibility. This is evidenced by the multiobjective approach.

Regarding the generation scenarios, it is found that the low generation levels in buses 1, 2 and 7 lead to infeasible operation for several obtained expansion plans. This information is important for reduction of scenarios by only considering the most critical ones, thus reducing computational effort.

4.5. Impacts of the MGS versus the basic planning scheme

To analyze the impact of the MGS on the traditional planning schemes, the following lines will address the problem of neglecting scenarios during the planning process, using as a reference the zero load shedding expansion plans for comparison purposes.

4.5.1. Garver system

When the basic planning problem modeled in (1)–(4), (8) is solved with generation rescheduling, the obtained plan has a cost of 110×10^3 USD with additions: $n_{3-5} = 1$ and $n_{4-6} = 3$ [39]. This expansion plan is only suitable under a unique generation scenario, but as discussed before, generation levels in each plant depend on different facts and a small change in generation levels, could lead to infeasibility. If this expansion plan is subject to operative analysis for the 4 feasible generation scenarios in Table 1, the resultant load shedding values are 300, 300, 120 and 38.54 MW, respectively.

It can be concluded from this data, that the basic planning scheme it not suitable for future operation of the power system, due to the multiple load patterns when generators are dispatched, which is the case in real life systems. This fact explains the importance of solving the TEP with MGS.

In addition, there is an evident increase in the cost of the expansion plan under MGS, which also justifies the importance of consid-

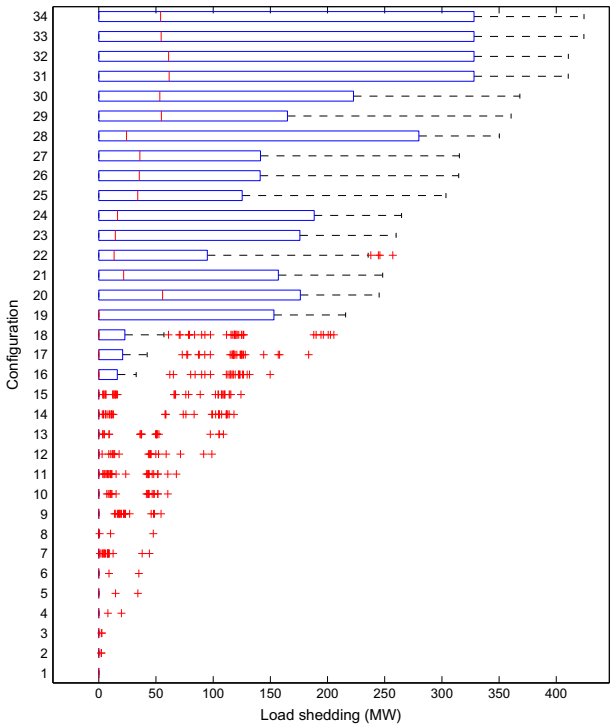

Fig. 8. Load shedding boxplot for IEEE 24-bus system with variable demand.

ering demand uncertainty to partially alleviate extra costs, as already shown for Garver system. Table 5 shows the trade-offs between the cost, load shedding, and supplied power, for each planning scheme. These trade-offs justify the multiobjective approach presented in this work, given that multiple expansion plans can be found and the extra-costs of considering MGS can be also mitigated, as already discussed.

4.5.2. IEEE 24-bus system

When only basic planning constraints are taken into account, the obtained expansion plan has a cost of 152×10^6 USD. This expansion plan is related to the following additions: $n_{06-10} = 1, n_{07-08} = 2, n_{10-12} = 1$ and $n_{14-16} = 1$ [26]. If the 178 generation scenarios are analyzed with these reinforcements, the minimum, average and maximum load shedding values are 144, 825 and 1488 MW respectively, which turns this expansion plan into infeasible. This clearly shows that including MGS in the analysis is necessary for a proper planning of the future network in order to face different generation levels.

Table 6 summarizes the main information for all planning schemes and the differences for the most important variables: cost, load shedding and supplied demand.

4.6. Computational performance of the proposed algorithm

To determine the competence of the enhanced algorithm, different tests were carried out for both networks and fixed demand cases. The performance of the proposed algorithm versus the basic NSGA-II, is measured with the number of PLs to achieve good quality solutions.

Table 6
Comparison for all planning schemes, IEEE 24-bus system.

Parameter	Generation (MW)		
	Basic	MGS with uncertainty	MGS w/o uncertainty
Cost ($\times 10^6$ USD)	152	1004	1330
Average load shedding under MGS (MW)	825	0	0
Supplied power	8550	8977.5	8550

4.6.1. Garver system

To carry out tests with the basic NSGA-II, the initialization scheme explained in sub Section 3.4.1 was used, and both multiobjective algorithms used a population of 50 individuals. Each algorithm was run 10 times and stopped when the 7 point Pareto front in Fig. 3 was found. For this test system, the Pareto front was obtained in all trials for both algorithms, but there was an important difference in the number of PLs they took to achieve the complete set of solutions. As depicted in Table 7, the basic NSGA-II takes in average, more than 6 times to find the Pareto front, when compared with the enhanced NSGA-II.

As explained before, the basic NSGA-II needs to calculate more PLs in order to achieve good quality results. This is explained by the fact that in each iteration, an offspring population Q of size NP is created, increasing the number of times the operative problem has to be solved, and also increasing the computational effort. On the contrary, the enhanced algorithm exploits the best of the Chu–Beasley logic, by keeping the population size constant, and reducing drastically the PLs to be calculated. Furthermore, the enhanced methodology controls diversity and assures that all expansion plans in the population are different, which increases the search capability of the algorithm, leading the solutions towards good quality regions, as demonstrated with the results.

Even comparing the best performance of the basic scheme with the worst for the enhanced methodology, the latter turns out to stand out, with 14500 PLs versus 45000. From these data, it is concluded that in 100% of the cases, the authors' proposal converges significantly faster to the optimal Pareto front.

4.6.2. IEEE 24-bus system

This test system presents a much more complex and demanding challenge than the previous one. Mainly for being a larger system, and more importantly, for having 178 scenarios to be evaluated, hence, the computational effort is critical due to the combinatorial explosion.

The algorithms were run 10 times each, and the enhanced methodology was able to find the 39 Pareto points in Fig. 5 for all trials, within a range of 10–14 million PLs. On the other hand, the basic NSGA-II losses Pareto-optimality capabilities, since none of the trials could find the entire set of 39 points. In addition, the algorithm was stopped after 100 million PLs, given that no

Table 5
Comparison for all planning schemes, Garver system.

Parameter	Generation (MW)		
	Basic	MGS with uncertainty	MGS w/o uncertainty
Cost ($\times 10^3$ USD)	110	238	268
Accumulated load shedding under MGS (MW)	758	0	0
Supplied power	760	798	760

Table 7
Comparison of algorithms to obtain the 7 point Pareto front, Garver system.

Algorithm	Parameters	Average PLs	Range of PLs	Standard dev.
Basic NSGA-II	$\rho_{div} = 5$, $\rho_{mut} = 4$	62,282	45,000–80,000	11,293
Enhanced NSGA-II	$\rho_{div} = 5$, $\rho_{mut} = 4$, $kk = 2$	9442	5500–14,500	2720

Table 8
Performance comparison for the basic and enhanced NSGA-II, IEEE 24-bus system.

Algorithm	Parameters	Average obtained points	Average PLs (M)
Basic NSGA-II	$\rho_{div} = 9$, $\rho_{mut} = 5$	26	100
Enhanced NSGA-II	$\rho_{div} = 9$, $\rho_{mut} = 5$, $kk = 2$	39	12.3

improvement was evident up to this point, and also because of computational effort being already prohibitive.

Table 8 shows the summary of the trials performed, the differences of PL computation and Pareto points obtained.

For all cases, the number of PLs with the enhanced NSGA-II is less when compared to the basic scheme and the number of plans is also higher, concluding that performance and optimality are improved. Again, this shows the advantages of the proposed method to handle NP-hard problems such as the TEP including MGS.

5. Conclusions

A multiobjective approach has been proposed to address the problem of TEP when MGS are taken into account. The model considers cost as one of the objective functions, and low levels of load shedding as objective function 2, both to be minimized.

A new multiobjective algorithm is proposed to solve the problem. This approach includes features of the NSGA-II and the CBGA, such as crowding distance, elitism, and diversity. This new algorithm allows to find a set of Pareto optimal expansion plans for fixed and variable demand. Solutions under these considerations are found for the 6-bus network proposed by Garver and the IEEE 24-bus test system. The proposed algorithm stands out over the basic NSGA-II, substantially improving computational effort and optimality.

When market constraints are considered, the cost of the expansion plan increases. The multiobjective approach returns a set of solutions with lower levels of cost and allows to identify potential savings that are not present under a single objective approach. The trade-off solutions are to be analyzed by a decision maker to select a plan with higher level information.

Pareto optimal plans are analyzed to obtain information for load shedding within the generation scenarios. This information shows that there are interesting plans to be considered, if other metrics are used to calculate objective function 2.

Future work can include comparison of these metrics and also generation scenario reduction by selecting the most critical and realistic ones. This could lead to decrease computational effort.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ijepes.2014.04.063.

References

[1] Torres SP, Castro CA. Parallel particle swarm optimization applied to the static transmission expansion planning problem. In: 2012 Sixth IEEE/PES transmission and distribution: Latin America conference and exposition (T D-LA); 2012. p. 1–6, http://dx.doi.org/10.1109/TDC-LA.2012.6319053.

[2] Silva I, Rider M, Romero R, Murari C. Genetic algorithm of Chu and Beasley for static and multistage transmission expansion planning. In: Power engineering society general meeting, 2006. IEEE; 2006. 7 pp., http://dx.doi.org/10.1109/PES.2006.1709172.

[3] Hemmati R, Hooshmand R-A, Khodabakhshian A. State-of-the-art of transmission expansion planning: comprehensive review. Renew Sustain Energy Rev 2013;23(0):312–9. http://dx.doi.org/10.1016/j.rser.2013.03.015. ISSN: 1364-0321.

[4] Escobar A, Gallego R, Romero R. Multistage and coordinated planning of the expansion of transmission systems. IEEE Trans Power Syst 2004;19(2):735–44. http://dx.doi.org/10.1109/TPWRS.2004.825920. ISSN: 0885-8950.

[5] Asada E, Carreno E, Romero R, Garcia A. A branch-and-bound algorithm for the multi-stage transmission expansion planning. Power Eng Soc General Meet, vol. 1. IEEE; 2005. p. 171–6. http://dx.doi.org/10.1109/PES.2005.1489071.

[6] Zhang H, Vittal V, Heydt G, Quintero J. A mixed-integer linear programming approach for multi-stage security-constrained transmission expansion planning. IEEE Trans Power Syst 2012;27(2):1125–33. http://dx.doi.org/10.1109/TPWRS.2011.2178000. ISSN: 0885-8950.

[7] da Rocha MC, Saraiva JT. A discrete evolutionary {PSO} based approach to the multiyear transmission expansion planning problem considering demand uncertainties. Int J Electr Power Energy Syst 2013;45(1):427–42. http://dx.doi.org/10.1016/j.ijepes.2012.09.009. ISSN: 0142-0615.

[8] Hinojosa V, Galleguillos N, Nuques B. A simulated rebounding algorithm applied to the multi-stage security-constrained transmission expansion planning in power systems. Int J Electr Power Energy Syst 2013;47(0):168–80. http://dx.doi.org/10.1016/j.ijepes.2012.10.052. ISSN: 0142-0615.

[9] Garver L. Transmission network estimation using linear programming. IEEE Trans Power Appar Syst, PAS-89 1970(7):1688–97. http://dx.doi.org/10.1109/TPAS.1970.292825. ISSN: 0018-9510.

[10] Romero R, Monticelli A, Garcia A, Haffner S. Test systems and mathematical models for transmission network expansion planning. IEE Proc – Gener Transm Distrib 2002;149(1):27–36. http://dx.doi.org/10.1049/ip-gtd:20020026. ISSN: 1350-2360.

[11] Sanchez I, Romero R, Mantovani J, Rider M. Transmission-expansion planning using the DC model and nonlinear-programming technique. IEE Proc – Gener Transm Distrib 2005;152(6):763–9. http://dx.doi.org/10.1049/ip-gtd:20050074. ISSN: 1350-2360.

[12] Rider M, Gallego L, Romero R, Garcia A. Heuristic algorithm to solve the short term transmission network expansion planning. In: Power engineering society general meeting, 2007. IEEE; 2007. p. 1–7. ISSN: 1932-5517.

[13] Akbari T, Rahimi-Kian A, Heidarizadeh M. Security-constrained transmission expansion planning: a multi-objective approach. In: 2011 19th Iranian conference on electrical engineering (ICEE); 2011. p. 1–1.

[14] Villasana R, Garver L, Salon S. Transmission network planning using linear programming. IEEE Trans Power Appar Syst, PAS-104 1985(2):349–56. http://dx.doi.org/10.1109/TPAS.1985.319049. ISSN: 0018-9510.

[15] Kim K, Park Y, Lee K. Optimal long term transmission expansion planning based on maximum principle. IEEE Trans Power Syst 1988;3(4):1494–501. http://dx.doi.org/10.1109/59.192958. ISSN: 0885-8950.

[16] Dusonchet Y, El-Abiad A. Transmission planning using discrete dynamic optimizing. IEEE Trans Power Appar Syst, PAS-92 1973(4):1358–71. http://dx.doi.org/10.1109/TPAS.1973.293543. ISSN: 0018-9510.

[17] Youssef H, Hackam R. New transmission planning model. IEEE Trans Power Syst 1989;4(1):9–18. http://dx.doi.org/10.1109/59.32451. ISSN: 0885-8950.

[18] Bahiense L, Oliveira G, Pereira M, Granville S. A mixed integer disjunctive model for transmission network expansion. IEEE Trans Power Syst 2001;16(3):560–5. http://dx.doi.org/10.1109/59.932295. ISSN: 0885-8950.

[19] Romero R, Monticelli A. A zero-one implicit enumeration method for optimizing investments in transmission expansion planning. IEEE Trans Power Syst 1994;9(3):1385–91. http://dx.doi.org/10.1109/59.336126. ISSN: 0885-8950.

[20] Binato S, Pereira MVF, Granville S. A new Benders decomposition approach to solve power transmission network design problems. IEEE Trans Power Syst 2001;16(2):235–40. http://dx.doi.org/10.1109/59.918292. ISSN: 0885-8950.

[21] Haffner S, Monticelli A, Garcia A, Mantovani J, Romero R. Branch and bound algorithm for transmission system expansion planning using a transportation

model. IEE Proc – Gener Transm Distrib 2000;147(3):149–56. http://dx.doi.org/10.1049/ip-gtd:20000337. ISSN: 1350-2360.

[22] Gallego R, Monticelli A, Romero R. Comparative studies on nonconvex optimization methods for transmission network expansion planning. IEEE Trans Power Syst 1998;13(3):822–8. http://dx.doi.org/10.1109/59.708680. ISSN: 0885-8950.

[23] Gallego R, Romero R, Monticelli A. Tabu search algorithm for network synthesis. IEEE Trans Power Syst 2000;15(2):490–5. http://dx.doi.org/10.1109/59.867130. ISSN: 0885-8950.

[24] Gallego RA. Planejamento a Longo Prazo de Sistemas de Transmissao Usando Tecnicas de Otimizacao Combinatorial. Ph.D. thesis, FEEC DSEE Unicamp, Brasil; 1997.

[25] Silva JMAEL, Gil HA. Transmission planning based on heuristic methods (en portugues). In: Proc. VII symp. specialists in elect. oper. expansion planning, Curitiba, Brazil; 2000.

[26] De J Silva I, Rider M, Romero R, Garcia A, Murari C. Transmission network expansion planning with security constraints. IEE Proc – Gener Transm Distrib 2005;152(6):828–36. http://dx.doi.org/10.1049/ip-gtd:20045217. ISSN: 1350-2360.

[27] Arroyo JM, Fernndez FJ. Application of a genetic algorithm to n-K power system security assessment. Int J Electr Power Energy Syst 2013;49(0):114–21. http://dx.doi.org/10.1016/j.ijepes.2012.12.011. ISSN: 0142-0615.

[28] Murugan P. Modified particle swarm optimisation with a novel initialisation for finding optimal solution to the transmission expansion planning problem. IET Gener Transm Distrib 2012;6(11):1132–42. http://dx.doi.org/10.1049/iet-gtd.2012.0183. ISSN: 1751-8687.

[29] Aguado J, de la Torre S, Contreras J, Conejo A, Martnez A. Market-driven dynamic transmission expansion planning. Electr Power Syst Res 2012;82(1):88–94. http://dx.doi.org/10.1016/j.epsr.2011.09.001. ISSN: 0378-7796.

[30] Foroud AA, Abdoos AA, Keypour R, Amirahmadi M. A multi-objective framework for dynamic transmission expansion planning in competitive electricity market. Int J Electr Power Energy Syst 2010;32(8):861–72. http://dx.doi.org/10.1016/j.ijepes.2010.01.027. ISSN: 0142-0615.

[31] Georgilakis PS. Market-based transmission expansion planning by improved differential evolution. Int J Electr Power Energy Syst 2010;32(5):450–6. http://dx.doi.org/10.1016/j.ijepes.2009.09.019. ISSN: 0142-0615.

[32] Gunnaasankaraan H, Viswanath A, Mahata K, Goel L. Transmission planning by minimizing curtailment of market transactions. Electr Power Syst Res 2013;101:1–8. http://dx.doi.org/10.1016/j.epsr.2013.02.010. ISSN: 03787796.

[33] Kamyab G-R, Fotuhi-Firuzabad M, Rashidinejad M. A PSO based approach for multi-stage transmission expansion planning in electricity markets. Int J Electr Power Energy Syst 2014;54:91–100. http://dx.doi.org/10.1016/j.ijepes.2013.06.027. ISSN: 01420615.

[34] Leou R-C. A multi-year transmission planning under a deregulated market. Int J Electr Power Energy Syst 2011;33(3):708–14. http://dx.doi.org/10.1016/j.ijepes.2010.11.020. ISSN: 01420615.

[35] Wang Y, Cheng H, Wang C, Hu Z, Yao L, Ma Z, et al. Pareto optimality-based multi-objective transmission planning considering transmission congestion. Electr Power Syst Res 2008;78(9):1619–26. ISSN: 0378-7796, doi: http://dx.doi.org/10.1016/j.epsr.2008.02.004.

[36] Fang R, Hill D. A new strategy for transmission expansion in competitive electricity markets. IEEE Trans Power Syst 2003;18(1):374–80. http://dx.doi.org/10.1109/TPWRS.2002.807083. ISSN: 0885-8950.

[37] Rider M, Silva I de J, Romero R, Garcia A, Murari C. Transmission network expansion planning in full open market considering security constraints. In: Power tech, 2005 IEEE Russia; 2005. p. 1–6. http://dx.doi.org/10.1109/PTC.2005.4524456.

[38] Escobar A, Romero R, Gallego R. Transmission network expansion planning considering multiple generation scenarios. In: Transmission and distribution conference and exposition: Latin America, 2008 IEEE/PES; 2008. p. 1–6. http://dx.doi.org/10.1109/TDC-LA.2008.4641804.

[39] Romero R, Monticelli A, Garcia A, Haffner S. Test systems and mathematical models for transmission network expansion planning. IEE Proc – Gener Transm Distrib 2002;149(1):27–36. http://dx.doi.org/10.1049/ip-gtd:20020026. ISSN: 1350-2360.

[40] Correa C, Bolanos R, Garces A. Environmental transmission expansion planning using non-linear programming and evolutionary techniques. In: 2012 IEEE International Symposium on Alternative energies and energy quality (SIFAE); 2012. p. 1–5. http://dx.doi.org/10.1109/SIFAE.2012.6478893.

[41] Baldick R, Kahn E. Transmission planning issues in a competitive economic environment. IEEE Trans Power Syst 1993;8(4):1497–503. http://dx.doi.org/10.1109/59.260951. ISSN: 0885-8950.

[42] Lu M, Dong Z, Saha T. A framework for transmission planning in a competitive electricity market. In: 2005 IEEE/PES transmission and distribution conference and exhibition: Asia and Pacific; 2005. p. 1–6. http://dx.doi.org/10.1109/TDC.2005.1547025.

[43] de la Torre S, Conejo A, Contreras J. Transmission expansion planning in electricity markets. IEEE Trans Power Syst 2008;23(1):238–48. http://dx.doi.org/10.1109/TPWRS.2007.913717. ISSN: 0885-8950.

[44] Deb K. Multi-objective genetic algorithms: Problem difficulties and construction of test problems. Evol Comput 1999;7(3).

[45] Deb K, Agarwal S, Pratap A, Meyarivan T. A fast and elitist multiobjective genetic algorithm: NSGA-II. Technical report, Indian Institute of Technology, Kanpur: Kanpur Genetic Algorithms Laboratory (KanGAL), 2000.

[46] Deb K, Pratap A, Agarwal S, Meyarivan T. A fast elitist multi-objective genetic algorithm: NSGA-II. IEEE Trans Evolution Comput 2002;6:182–97.

[47] Deb K. Multi-objective optimization using evolutionary algorithms. John Wiley and sons; 2001.

[48] Romero R, Monticelli A. A hierarchical decomposition approach for transmission network expansion planning. IEEE Trans Power Syst 1994;9(1):373–80. http://dx.doi.org/10.1109/59.317588. ISSN: 0885-8950.

[49] H. Khorasani, M. Pourakbari-Kasmaei, R. Romero, Transmission expansion planning via a constructive heuristic algorithm in restructured electricity industry. In: 2013 3rd International conference on electric power and energy conversion systems (EPECS); 2013. p. 1–6. http://dx.doi.org/10.1109/EPECS.2013.6712989.