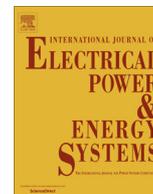




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## Lagrangian relaxation hybrid with evolutionary algorithm for short-term generation scheduling



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### ABSTRACT

Short-term generation scheduling is an important function in daily operational planning of power systems. It is defined as optimal scheduling of power generators over a scheduling period while respecting various generator constraints and system constraints. Objective of the problem includes costs associated with energy production, start-up cost and shut-down cost along with profits. The resulting problem is a large scale nonlinear mixed-integer optimization problem for which there is no exact solution technique available. The solution to the problem can be obtained only by complete enumeration, often at the cost of a prohibitively computation time requirement for realistic power systems. This paper presents a hybrid algorithm which combines Lagrangian Relaxation (LR) together with Evolutionary Algorithm (EA) to solve the problem in cooperative and competitive energy environments. Simulation studies were carried out on different systems containing various numbers of units. The outcomes from different algorithms are compared with that from the proposed hybrid algorithm and the advantages of the proposed algorithm are briefly discussed.

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### Introduction

Daily load patterns of many utilities exhibit variation between peak and off-peak hours due to less electricity usage on weekends than on weekdays, and at a lower rate between midnight and early morning than during the day. If sufficient generation is kept online throughout the day, some of the units will be operating near to their minimum generating limits during the off-peak period. This is not an economical solution. Therefore, the problem confronting the system operator is to determine which units should be taken offline during the off-peak period, and which units should be kept online. In most of the interconnected power systems, the power requirement is principally met by thermal power generation. Several operating strategies are possible to meet the required power demand, which varies from hour to hour throughout the day. Moreover, in order to supply high-quality electric power to customers in a secure and economic manner, Unit Commitment (UC) [1] is considered to be one of the best available options. It is thus recognized that the optimal unit commitment, which is the problem of determining the schedule of generating units within a power system, subject to device and operating constraints results

in substantial savings for electric utilities. The general objective of the unit commitment problem is to minimize total operating cost of system while satisfying all of the constraints so that a given security level can be met. Unit commitment problem is one of two linked optimization tasks of generation scheduling problem, which decides ON/OFF statuses of generators over the scheduling period. Economic Dispatch (ED) problem is the other linked optimization problem that finds operating power levels of the committed generators.

The electric power industry has been using several efficient methodologies [2] for many years to solve the generation scheduling problem. However, as the power industry undergoes restructuring [3], the role of generation scheduling models is changing. Meanwhile, in modern restructured environments, small improvements in solutions can result significantly in the electricity market [3,4]. Further, deregulation of power systems has motivated small sized production units and distribution generation. As a result, modern power system contains a large number of generating units. As the number of units increases, the generation scheduling problem grows exponentially and needs excessive computation time and effort to solve. The solution to the generation scheduling problem can be obtained only by complete enumeration, often at the cost of a prohibitively large computation time requirements for medium sized power systems. Therefore, research endeavours

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**Nomenclature**

$a_i, b_i$ & $c_i$	fuel cost coefficients of unit $i$	$q$	dual value of Lagrangian function
$C$	operating cost	$r$	uncertainty in reserve or estimated probability of called reserve
CBUC	cost based unit commitment	$Re$	revenue
$DR_i$	ramp down rate of generation unit $i$	$R_i(t)$	allocated reserve power of generation unit $i$ at time $t$
EA	evolutionary algorithm	$RP(t)$	forecasted spinning reserve price at time $t$
ED	economic dispatch	$SP(t)$	forecasted spot market price at time $t$
$F_i(t)$	profit of generation unit $i$ at time $t$	$ST_{ci}$	cold start-up cost
$F_i(P_i(t))$	fuel cost of unit $i$ at time $t$	$ST_{hi}$	hot start-up cost
$F_{max}$	maximum of $[F_r]$	$ST_i(t)$	start-up cost of generation unit $i$ at time $t$
$F_r$	inverse of relative duality gap	$T$	number of scheduling time slots in the scheduling period
$I_i(t)$	ON/OFF status of generation unit $i$ at time $t$ (ON = 1 and OFF = 0)	$T_{i,cold}$	cold start-up time of generation unit $i$
$J$	primal value of Lagrangian function	$T_{i,Down}$	minimum down-time of generation unit $i$
$K$	a scaling factor	$T_{i,off}(t)$	Continuous off-time of generation unit $i$ at time $t$
LR	Lagrangian relaxation	$T_{i,on}(t)$	continuously on-time of generation unit $i$ at time $t$
LREA	Lagrangian relaxation – evolutionary algorithm	$T_{i,up}$	minimum up time of generation unit $i$
$N$	number of generation units	$TP$	total profit
PBUC	profit-based unit commitment	UC	unit commitment
$P_i(t)$	power generation of unit $i$ at time $t$	$UR_i$	ramp-up rate of generation unit $i$
$P_i^{max}(t)$	maximum generation limit of generation unit $i$ at time $t$	$\varepsilon$	relative duality gap
$P_i^{min}(t)$	minimum generation limit of generation unit $i$ at time $t$	$\lambda_t$ & $\mu_t$	Lagrangian multipliers
$P_t(t)$	system electricity demand at time $t$		
$P_R(t)$	system spinning reserve at time $t$		

have been focused to develop near-optimal generation scheduling algorithms which can be applicable to modern power systems.

A literature survey on the unit commitment methodologies reveals that various numerical optimization techniques have been employed to the problem. Specifically, there are traditional mathematical methodologies such as Priority List (PL) [2,3], Dynamic Programming (DP) [4], Branch and Bound (BB) [5] methods, Mixed-Integer Programming (MIP) [6] and Lagrangian Relaxation (LR) [7,8] methods. Among these, priority list is a simple and fast method but the quality of the solution is generally poor. Dynamic programming is flexible but the main disadvantage is the curse of dimensionality which increases in computational time. The shortcoming of branch and bound method is the exponential growth in the execution time with the size of unit commitment problem. Mixed-integer programming method for solving unit commitment problem fails when the number of units increases because that requires a large memory and suffers from great computational delay. Lagrangian relaxation method is used widely to solve unit commitment problem in the power industry. Even though Lagrangian relaxation method provides a fast solution, it may suffer from numerical convergence and solution quality. Recently, solution methodologies based on meta-heuristics such as Genetic Algorithm (GA) [9–12], Evolutionary Programming (EP) [13], Fuzzy Logic (FL), Artificial Neural Network (ANN), Simulated Annealing (SA) and swarm algorithms: Particle Swarm Optimization (PSO) [14–17] and Ant Colony Optimization (ACO) have also shown more promising results. These meta-heuristic optimization methodologies attract much attention because of their ability to search not only local optimal solutions but also global optimal solution and their ability to deal with various difficult nonlinear constraints. However, these meta-heuristic methods require a considerable amount of computational time to find the near-global optimum solution especially for large-scale unit commitment problems. Very recently, researchers show potential of solving with hybrid techniques [18–20]. These techniques that are combinations of the above methodologies, give even better solutions than that from above methodologies.

Even though, Lagrangian relaxation method is a widely accepted algorithm in real power system scheduling, and can

provide a fast solution, quality of solutions strongly depends on the algorithm used to update the Lagrangian multipliers. Typically, Lagrangian multipliers are updated by sub-gradient methods with a scaling factor. This paper presents a hybrid algorithm, LREA which uses Lagrangian relaxation together with evolutionary algorithm for solving short-term generation scheduling in cooperative and competitive environments. This hybrid methodology overcomes well-known several disadvantages of traditional sub-gradient methods and other disadvantages of modern computational algorithms such as large simulation time.

The remaining paper is organized as follows. Section ‘Problem formulation’ formulates the generation scheduling problem in both cooperative and competitive environments. Section ‘Proposed methodology: Lagrangian Relaxation Hybrids with Evolutionary Algorithm (LREA)’ proposes LREA for solving the generation scheduling problems. Section ‘Simulation studies and results’ provide the details of the simulation studies and simulation outcomes. Section ‘Conclusion’ concludes the paper.

**Problem formulation**

Generation scheduling problem involves the determination of states of power generating units for each time slot and power settings of the committed generating units subject to system constraints and generating unit constraints. Objective of generation scheduling problems varies based on the energy environments. This paper studies generation scheduling problems in both cooperative and competitive environments.

**Cost-based unit commitment problem**

Traditionally, in a vertically integrated utility which operates in cooperative energy environment, the system operator who has the knowledge of system components, constraints and operating costs of generating units makes decisions on generation scheduling. Cost-Based Unit Commitment [7,21] determines generating unit schedules for a utility by minimizing the operating cost and satisfying the prevailing system and units constraints over the

scheduling period. This unit commitment problem can be mathematically formulated as follows.

Minimize

$$\text{Total Operating Cost (TOC)} = \sum_{i=1}^N \sum_{t=1}^T [I_i(t) \cdot F_i(P_i(t)) + ST_i(t) \cdot (1 - I_i(t-1)) \cdot I_i(t)] \quad (1)$$

Fuel cost of unit  $i$  at time  $t$  is given by the following equation:

$$F_i(P_i(t)) = a_i + b_i P_i(t) + c_i P_i(t)^2 \quad (2)$$

Start-up cost of generation unit  $i$  at time  $t$  is given by following equation.

$$ST_i(t) = \begin{cases} ST_{hi} & \text{if } T_{i,off}(t) \leq T_{i,Down} + T_{i,cold} \\ ST_{ci} & \text{if } T_{i,off}(t) > T_{i,Down} + T_{i,cold} \end{cases} \quad (3)$$

This minimization problem is subject to following system and unit constraints.

1. System real power balance

$$\sum_{i=1}^N I_i(t) \cdot P_i(t) = P_l(t) \quad (4)$$

2. System spinning reserve requirement

$$\sum_{i=1}^N I_i(t) \cdot P_i^{max} \geq P_l(t) + P_R(t) \quad (5)$$

3. Generation unit's limits

$$P_i^{min}(t) \leq P_i(t) \leq P_i^{max}(t) \quad (6)$$

4. Minimum up and minimum down times

$$(T_{i,on}(t-1) - T_{i,Up}) \cdot (I_i(t-1) - I_i(t)) \geq 0 \quad (7)$$

$$(T_{i,off}(t-1) - T_{i,Down}) \cdot (I_i(t-1) - I_i(t)) \geq 0$$

5. Ramp up and ramp down rates

$$P_i(t) - P_i(t-1) \leq UR_i \quad (8)$$

$$P_i(t-1) - P_i(t) \leq DR_i$$

### Profit-based unit commitment problem

In a restructured system [22,23], generation companies sell power in power market and sell reserve in the ancillary market. These markets provide competitive energy environments for utilities and customers. The amount of power and reserve sold depends on the way reserve payments are made. There are several ways available for reserve payment method in the world such as payment for power delivered, and payment for reserve allocated. In payment for power delivered, reserve is paid when only reserve is actually used. Therefore, the reserve price is higher than the power spot price; whereas, in payment for reserve allocated, generation companies receive the reserve price per unit of reserve for every time period that the reserve is allocated and not used. When the reserve is used, generation companies receive the spot price for the reserve that is generated. In this method, reserve price is much lower than the spot price.

In this paper, the power and reserve are sold based on the way reserve payment for power delivered [22]. In this case, profit based generation scheduling problem handled by generation companies under a competitive environment can be formulated mathematically as follows.

$$\text{Total Profit (TP)} = \text{Revenue (Re)} - \text{Operating Cost (C)} \quad (9)$$

Maximize

$$TP = \sum_{i=1}^N \sum_{t=1}^T F_i(t) \quad (10)$$

or

Minimize

$$-\sum_{i=1}^N \sum_{t=1}^T F_i(t) \quad (11)$$

The revenue (Re) from the spot energy and reserve is given by the following equation.

$$Re = \sum_{i=1}^N \sum_{t=1}^T P_i(t) \cdot SP(t) \cdot I_i(t) + \sum_{i=1}^N \sum_{t=1}^T r \cdot RP(t) \cdot R_i(t) \cdot I_i(t) \quad (12)$$

The production cost and start-up cost (C) is calculated by the following equation.

$$C = (1 - r) \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) \cdot I_i(t) + r \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t) + R_i(t)) \cdot I_i(t) + ST_i \cdot (1 - I_i(t-1)) \cdot I_i(t) \quad (13)$$

This optimization problem is subject to following system and unit constraints.

1. System energy constraints,

$$\sum_{i=1}^N P_i(t) \cdot I_i(t) \leq P_l(t) \quad (14)$$

2. System reserve constraints,

$$\sum_{i=1}^N R_i(t) \cdot I_i(t) \leq P_R(t) \quad (15)$$

3. Unit power and reserve limits,

$$0 \leq R_i(t) \leq P_i^{max}(t) - P_i^{min}(t) \quad (16)$$

$$R_i(t) + P_i(t) \leq P_i^{max}(t) \quad (17)$$

4. Unit minimum UP/Down durations

5. Unit ramping constraints

In restructured power markets [22,23], generation companies sell power in the spot market and sell spinning and non-spinning reserves in the reserve markets. As explained above, reserve payments can be made in different scenarios. In this paper, reserve is paid only when actually used. Therefore, the reserve price is higher than the spot price. The probability that the reserves are called and generated is also considered in the formulation to represent the uncertainty in reserves. Further, it is considered that there is no non-spinning reserve in the reserve market. It can be included in the problem formulation easily with another uncertainty constant with it. If the power and reserve are sold based on the way payment for reserve allocated, the formulation of profit based generation scheduling problem will be different in Eqs. (12) and (13). These equations will be replaced by Eqs. (18) and (19) respectively.

The revenue (Re) from the spot energy and reserve is given by the following equation.

$$Re = \sum_{i=1}^N \sum_{t=1}^T P_i(t) \cdot SP(t) \cdot I_i(t) + \sum_{i=1}^N \sum_{t=1}^T ((1 - r) \cdot RP(t) + r \cdot SP(t)) \cdot R_i(t) \cdot I_i(t) \quad (18)$$

The production cost and start-up cost (C) can be calculated by the following equation.

$$C = (1 - r) \cdot \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) \cdot I_i(t) + r \cdot \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t) + R_i(t)) \cdot I_i(t) + ST_i \cdot (1 - I_i(t-1)) \cdot I_i(t) \quad (19)$$

### Proposed methodology: Lagrangian Relaxation Hybrids with Evolutionary Algorithm (LREA)

The principal advantage of applying Lagrangian relaxation is its computational efficiency. The execution time of Lagrangian relaxation will increase linearly with the size of the problem [1,7,8]. The Lagrangian relaxation method allows the utilization of parallel computing techniques for single unit commitment sub problems with a small CPU time. However, Lagrangian relaxation decomposition procedure is dependent on the initial estimates of the Lagrangian multipliers, and on the method used to update the multipliers. In order to obtain a near-optimal solution, Lagrange multiplier adjustments are needed to be managed skilfully. In addition, the Lagrangian relaxation method often encounters difficulties as more complicated constraints are considered. The inclusion of a large number of multipliers could result in an optimization problem that is more difficult and even impossible to solve as the number of constraints grows and various heuristics are embedded in the Lagrangian relaxation algorithm.

Evolutionary Algorithm (EA) [9–11] is a general purpose stochastic and parallel search method, which can be used as an optimization technique for obtaining near-global optimum solutions of generation scheduling problems. This algorithm is inspired from genetics and evolution theories of natural selection and survival of the fittest. It is iterative procedure acting on a population of chromosomes, each chromosome being the encoding of a candidate solution to the problem. A fitness which depends on how well it solves the problem is associated with each chromosome. The objective function involves penalty term to penalize those potential solutions in which the problem constraints are not fulfilled. The objective function translates into a fitness which determines the solution's ability to survive and produce offspring. New generations of solutions are obtained by a process of selection, cross-over, and mutation. During the evolution process, new generations should give increasingly fitter solution and evolve toward an optimal solution.

In this paper, a hybrid algorithm (i.e. similar to the papers [22,23]) which is combined Lagrangian Relaxation (LR) and Evolutionary Algorithm (EA) is proposed to solve the concern problem. This methodology incorporates evolutionary algorithms into Lagrangian relaxation to update the Lagrangian multipliers and improve the performance of Lagrangian relaxation method. The evolutionary algorithms combine the adaptive nature of the natural genetics or the evolution procedures of organs with functional optimizations. The proposed LREA method consists of a two-stage cycle. The first stage is to search for the constrained minimum of Lagrangian function under constant Lagrangian multipliers by two-state dynamic programming. The second stage is to maximize the Lagrangian function with respect to the Lagrangian multipliers adjusted by evolutionary algorithms. The overall procedure of the proposed method is described in Fig. 1.

In the proposed methodology, updating of the Lagrange multipliers is done by an evolutionary algorithm. Components of the evolutionary algorithm are described as follows.

A population of chromosomes was constructed as below, and initialised uniformly randomly.

For  $T$  time slots in the scheduling periods, an array of control variable  $\lambda$  and  $\mu$  vectors are represented as genes in the chromosomes. This can be shown as below.

$$\lambda = [\lambda_1 \lambda_2 \dots \lambda_T]$$

$$\mu = [\mu_1 \mu_2 \dots \mu_T] \quad (20)$$

Fitness function for evolutionary algorithm is chosen as follows.

$$\text{Fitness} = \frac{1}{1 + K \left( \frac{F_{\max}}{F_r} - 1 \right)} \quad (21)$$

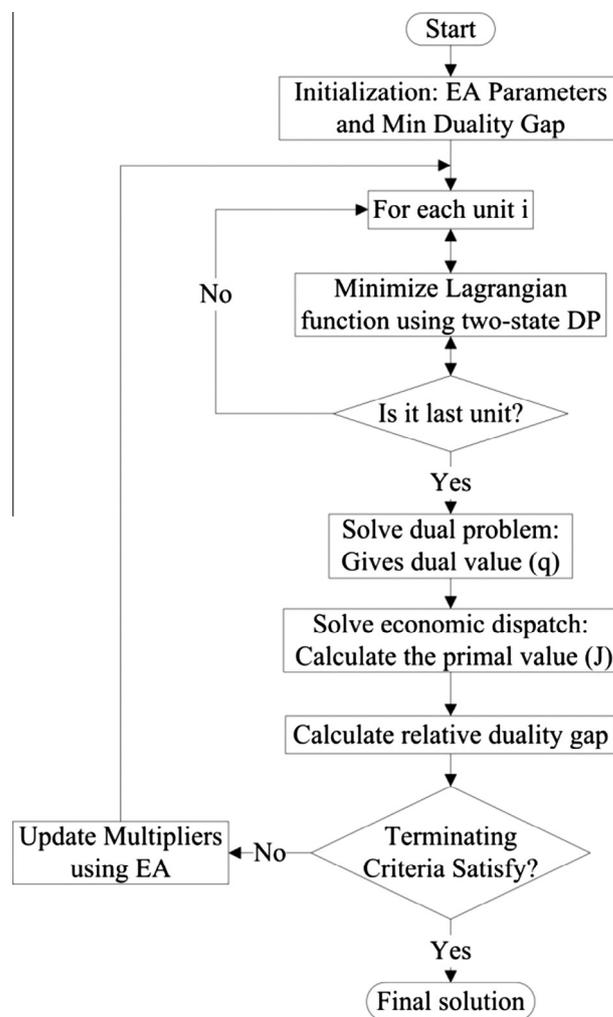


Fig. 1. LREA for unit commitment problem.

where,

$$F_r = \frac{1}{\varepsilon} \quad (22)$$

$$\varepsilon = \frac{J - q}{q} \quad (23)$$

During the simulation of the algorithm, new population of chromosomes is produced from the existing population by adding Gaussian random number with zero mean and a predefined standard deviation to each individual. Tournament method is used as the selection technique. The duality gap that is the difference between primal and dual problem is used as a terminating criteria for the algorithm.

Except the methodology for updating of Lagrange multipliers, the proposed methodology is the same as the traditional Lagrangian relaxation methodology. This is given in the following section.

#### Lagrangian relaxation for generation scheduling

The Lagrangian function for generation scheduling in a cooperative energy environment can be formulated as follows.

$$L(P, R, \lambda, \mu) = J(P, R, I) + \sum_{t=1}^T \lambda_t \left( P_I(t) - \sum_{i=1}^N P_i(t) \cdot I_i(t) \right) + \sum_{t=1}^T \mu_t \left( P_I(t) + P_R(t) - \sum_{i=1}^N P_i^{\max}(t) \cdot I_i(t) \right) \quad (24)$$

where

$$J(P, R, I) = \sum_{i=1}^N \sum_{t=1}^T [I_i(t) \cdot F_i(P_i(t)) + S_i(t) \cdot (1 - I_i(t - 1)) \cdot I_i(t)] \quad (25)$$

Then,

$$L(P, R, \lambda, \mu) = \sum_{i=1}^N \sum_{t=1}^T [I_i(t) \cdot F_i(P_i(t)) + S_i(t) \cdot (1 - I_i(t - 1)) \cdot I_i(t)] + \sum_{t=1}^T \lambda_t \left( P_I(t) - \sum_{i=1}^N P_i(t) \cdot I_i(t) \right) + \sum_{t=1}^T \mu_t \left( P_I(t) + P_R(t) - \sum_{i=1}^N P_i^{max}(t) \cdot I_i(t) \right) \quad (26)$$

In case of a competitive energy environment, the Lagrangian function for generation scheduling can be formulated as follows.

$$L(P, R, \lambda, \mu) = J(P, R, I) + \sum_{t=1}^T \lambda_t \left( P_I(t) - \sum_{i=1}^N P_i(t) \cdot I_i(t) \right) + \sum_{t=1}^T \mu_t \left( SR - \sum_{i=1}^N R_i(t) \cdot I_i(t) \right) \quad (27)$$

where

$$J(P, R, I) = (1 - r) \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) \cdot I_i(t) + r \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) + R_i(t) \cdot I_i(t) + ST_i \cdot (1 - I_i(t - 1)) \cdot I_i(t) - \sum_{i=1}^N \sum_{t=1}^T (P_i(t) \cdot SP(t)) \cdot I_i(t) - \sum_{i=1}^N \sum_{t=1}^T r \cdot RP(t) \cdot R_i(t) \cdot I_i(t) \quad (28)$$

Then,

$$L(P, R, \lambda, \mu) = (1 - r) \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) \cdot I_i(t) + r \sum_{i=1}^N \sum_{t=1}^T F_i(P_i(t)) + R_i(t) \cdot I_i(t) + ST_i \cdot (1 - I_i(t - 1)) \cdot I_i(t) - \sum_{i=1}^N \sum_{t=1}^T (P_i(t) \cdot SP(t)) \cdot I_i(t) - \sum_{i=1}^N \sum_{t=1}^T r \cdot RP(t) \cdot R_i(t) \cdot I_i(t) + \sum_{t=1}^T \lambda_t \left( P_I(t) - \sum_{i=1}^N P_i(t) \cdot I_i(t) \right) + \sum_{t=1}^T \mu_t \left( SR - \sum_{i=1}^N R_i(t) \cdot I_i(t) \right) \quad (29)$$

The Lagrangian relaxation procedure solves the unit commitment problem by “relaxing” or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through the dual optimization procedure which attempts to reach the constrained optimum by maximizing the Lagrangian function with respect to the Lagrangian multipliers and minimizing the function with respect to the control variables  $I, P,$  and  $R$ .

$$q^* = \max_{\lambda, \mu} q(\lambda, \mu) \quad (30)$$

where

$$q(\lambda, \mu) = \min_{I, P, R} L(P, R, \lambda, \mu, I) \quad (31)$$

Subject to the constraints given in the problem formations.

Fig. 2 shows a flowchart of Lagrangian relaxation methodology for unit commitment problems. The detail of the algorithm is described step by step as in details as follows.

The Lagrange function can be rewritten by dropping the constant terms as follows.

For cooperative environment,

$$L = \sum_{i=1}^N \left[ \sum_{t=1}^T \{ F_i(P_i(t)) + ST_i \cdot (1 - I_i(t - 1)) - \lambda_t \cdot P_i(t) - \mu_t \cdot R_i(t) \} \cdot I_i(t) \right] \quad (32)$$

For competitive environment,

$$L = \sum_{i=1}^N \left[ \sum_{t=1}^T \{ (1 - r) \cdot F_i(P_i(t)) + r \cdot F_i(P_i(t) + R_i(t)) + ST_i \cdot (1 - I_i(t - 1)) - P_i(t) \cdot SP(t) - r \cdot RP(t) \cdot R_i(t) + \lambda_t \cdot P_i(t) + \mu_t \cdot R_i(t) \} \cdot I_i(t) \right] \quad (33)$$

Term inside the bracket of Eqs. (31) or (32) can be solved separately for each generating unit without regard for what is happening on the other generating units.

The minimum of the Lagrange function is found by solving for the minimum for each generating unit over all time periods. Dynamic programming is used to solve this problem. The dynamic programming determines the optimal schedule of each unit over the scheduling time period. More specifically, for each state in each hour, the ON/OFF decision making is needed to select a solution that would result in a higher profit/lower operating cost by comparing the combination of the start-up cost and the accumulated

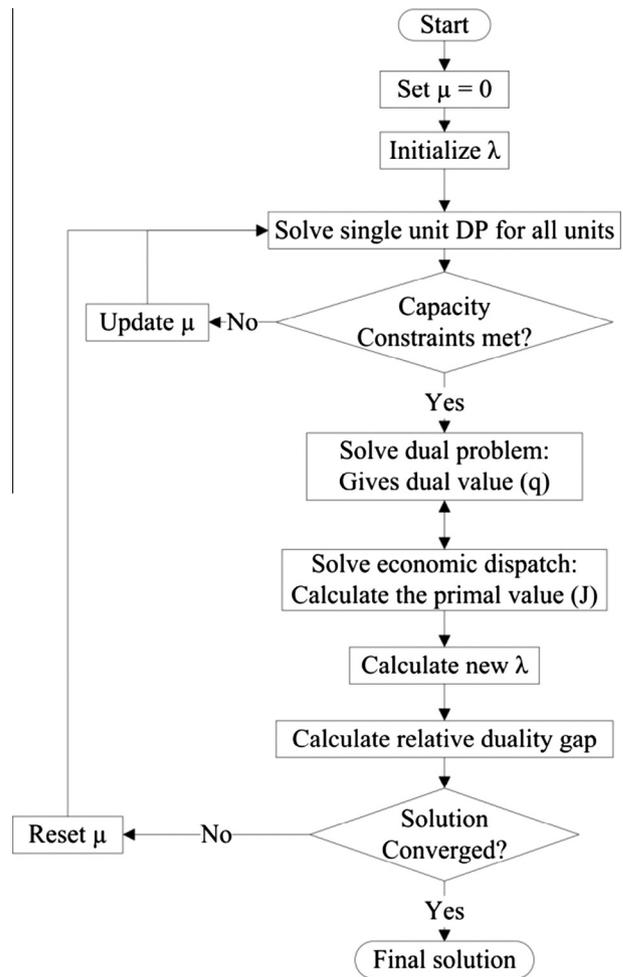


Fig. 2. Lagrangian relaxation for unit commitment problem.

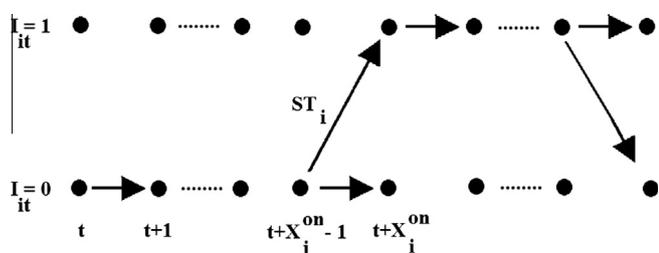


Fig. 3. Search paths of each unit in dynamic programming.

profits from two historical routes. This procedure is shown in Fig. 3.

At this stage calculate the dual value of the problem. Then, in order to maximize the Lagrange function, Lagrange multipliers must be suitably adjusted. Traditionally, it is done by a gradient

based approach. Once the new Lagrange multipliers are known, power and reserve settings of each committed units can be found by Economic Dispatch (ED) [1].

The economic dispatch of generators is a key element in the optimal operation of power generation systems. The main goal is the generation of a given amount of electricity at the highest profit/least operating cost possible. Once the unit commitment status is determined, an economic dispatch problem is formulated and solved to ensure the feasibility of the original unit commitment solution. The economic dispatch problem at time is given as follows.

For cooperative environment

$$\text{Minimize} \sum_{i=1}^N J(I, P) \quad (34)$$

Table 1 Data of 10-unit system.

Parameters	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
$P_{max}$ (MW)	455	455	130	130	162	80	85	55	55	55
$P_{min}$ (MW)	150	150	20	20	25	20	25	10	10	10
$R_{max}$ (MW)	45.5	45.5	13.0	13.0	16.2	8.0	8.5	5.5	5.5	5.5
$R_{min}$ (MW)	0	0	0	0	0	0	0	0	0	0
$a$ (\$/h)	1000	970	700	680	450	370	480	660	665	670
$b$ (\$/MW h)	16.19	17.26	16.60	16.50	19.70	22.26	27.74	25.92	27.27	27.79
$c$ (\$/MW <sup>2</sup> h) $\times 10^{-4}$	4.8	3.1	20	21.1	39.8	71.2	7.9	41.3	22.2	17.3
Min up time (h)	8	8	5	5	6	3	3	1	1	1
Min down time (h)	8	8	5	5	6	3	3	1	1	1
Hot start-up cost (\$)	4500	5000	550	560	900	170	260	30	30	30
Cold start-up cost (\$)	9000	10,000	1100	1120	1800	340	520	60	60	60
Cold start-up hrs (h)	5	5	4	4	4	2	2	0	0	0
Initial status (h)	8	8	-5	-5	-6	-3	-3	-1	-1	-1

Table 2 Forecasted demands, spinning reserves and spot market prices for 10-unit system.

Hour	Demand (MW)	Reserve (MW)	Spot price (\$/MW h)	Hour	Demand (MW)	Reserve (MW)	Spot price (\$/MW h)
1	700	70	22.15	13	1400	140	24.60
2	750	75	22.00	14	1300	130	24.50
3	850	85	23.10	15	1200	120	22.50
4	950	95	22.65	16	1050	105	22.30
5	1000	100	23.25	17	1000	100	22.25
6	1100	110	22.95	18	1100	110	22.05
7	1150	115	22.50	19	1200	120	22.20
8	1200	120	22.15	20	1400	140	22.65
9	1300	130	22.80	21	1300	130	23.10
10	1400	140	29.35	22	1100	110	22.95
11	1450	145	30.15	23	900	90	22.75
12	1500	150	31.65	24	800	80	22.55

Table 3 Simulation results obtained for cooperative energy environment.

No of units	GA		LR		LREA	
	Average cost (\$)	Elapsed time (s)	Average cost (\$)	Elapsed time (s)	Average cost (\$)	Elapsed time (s)
10	565,825	823	565,825	117	565,825	1232
20	1,126,249	1443	1,120,660	210	1,120,660	1301
40	2,251,981	2411	2,258,510	382	2,248,700	1352
60	3,383,631	3912	3,394,056	592	3,367,902	1501
80	4,504,989	5181	4,526,043	890	4,492,012	1981
100	5,587,538	8348	5,657,290	1312	5,510,320	2568
150	8,405,341	10,750	8,667,823	1603	8,400,123	3023
200	11,175,072	16,690	11,304,567	2102	11,110,343	4234
300	16,762,600	24,043	17,002,344	3201	16,053,852	5983
400	22,350,152	33,081	23,000,234	4322	21,943,688	6502
500	27,937,680	40,001	29,023,445	5023	26,707,993	7034

**Table 4**  
Power setting of each unit of 10 unit system in cooperative energy environment.

Hour	Power settings (MW)									
	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
1	455	245	0	0	0	0	0	0	0	0
2	455	295	0	0	0	0	0	0	0	0
3	455	370	0	0	25	0	0	0	0	0
4	455	455	0	0	40	0	0	0	0	0
5	455	390	0	130	25	0	0	0	0	0
6	455	360	130	130	25	0	0	0	0	0
7	455	410	130	130	25	0	0	0	0	0
8	455	455	130	130	30	0	0	0	0	0
9	455	455	130	130	85	20	25	0	0	0
10	455	455	130	130	162	33	25	10	0	0
11	455	455	130	130	162	73	25	10	10	0
12	455	455	130	130	162	80	25	43	10	10
13	455	455	130	130	162	33	25	10	0	0
14	455	455	130	130	85	20	25	0	0	0
15	455	455	130	130	30	0	0	0	0	0
16	455	310	130	130	25	0	0	0	0	0
17	455	260	130	130	25	0	0	0	0	0
18	455	360	130	130	25	0	0	0	0	0
19	455	455	130	130	30	0	0	0	0	0
20	455	455	130	130	162	33	25	10	0	0
21	455	455	130	130	85	20	25	0	0	0
22	455	455	0	0	145	20	25	0	0	0
23	455	420	0	0	25	0	0	0	0	0
24	455	345	0	0	0	0	0	0	0	0

**Table 5**  
Power setting of each unit of 10 unit system in competitive energy environment.

Hour	Power settings (MW)/reserve settings (MW)									
	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
1	455/0	245/70	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
2	455/0	295/75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
3	455/0	395/60	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
4	455/0	455/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
5	455/0	455/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
6	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
7	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
8	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
9	455/0	455/0	130/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
10	455/0	455/0	130/0	130/0	162/0	68/12	0/0	0/0	0/0	0/0
11	455/0	455/0	130/0	130/0	162/0	80/0	0/0	0/0	0/0	0/0
12	455/0	455/0	130/0	130/0	162/0	80/0	0/0	0/0	0/0	0/0
13	455/0	455/0	130/0	130/0	162/0	0/0	0/0	0/0	0/0	0/0
14	455/0	455/0	130/0	130/0	130/32	0/0	0/0	0/0	0/0	0/0
15	455/0	455/0	130/0	130/0	160/2	0/0	0/0	0/0	0/0	0/0
16	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
17	455/0	415/40	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
18	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
19	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
20	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
21	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
22	455/0	455/0	0/0	130/0	0/0	0/0	0/0	0/0	0/0	0/0
23	455/0	445/10	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
24	455/0	345/80	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0

For competitive environment

Minimize 
$$\sum_{i=1}^N -F_i(t) \tag{35}$$

Subject to energy requirement, reserve requirement and generation unit limits.

Typical Lambda-iteration method [1] is used to solve the economic dispatch problem.

The iteration continuous until it meets the convergence criteria. Here the relative duality gap is used as the convergence criteria.

Relative Duality Gap = 
$$\frac{J - q}{q} \tag{36}$$

**Simulation studies and results**

Initially, some simulation studies [21,22] were carried out on test systems for benchmarking of the proposed algorithm. Then the feasibility of the proposed algorithm is demonstrated on power systems with much larger number of units.

*Cooperative environment*

The feasibility of the proposed algorithms is demonstrated with different size of power systems containing 10–500 units and the test results are compared with each other in terms of solution quality and convergence characteristic. Characteristics of units and load for the 10-units system are given in Tables 1 and 2.

**Table 6**  
Profits obtained for competitive energy environment.

No of units	GA		LR		LREA	
	Average profit (\$)	Elapsed time (s)	Average profit (\$)	Elapsed time (s)	Average profit (\$)	Elapsed time (s)
10	107,872	752	107,872	111	107,875	1112
20	215,621	1323	215,743	198	215,747	1230
40	431,361	2321	431,223	368	431,489	1365
60	644,676	3821	642,122	571	647,235	1498
80	837,729	5099	812,499	851	862,979	1954
100	1,038,262	8235	997,924	1298	1,078,718	2537
150	1,577,493	11,243	1,528,782	2046	1,678,332	3021
200	2,075,654	16,345	2,000,234	3294	2,164,749	4364
300	3,115,765	23,654	3,109,456	4594	3,278,398	5868
400	4,156,760	31,456	4,098,234	5987	4,387,573	7365
500	5,209,568	42,497	5,198,374	7943	5,457,975	9023

Spinning reserve of the system is assumed to be 10% of the load. The different number of units systems is created by repeating each unit of the original problem by respective factors. Thus, test problems with different numbers units are obtained. Hourly load and reserve amounts are also scaled by the same factors.

After proper tuning the parameters of the proposed algorithm, simulation studies were carried out. The best results obtained by the algorithm are tabulated in Table 3. Further, Table 4 shows the power settings of each unit in case of 10 unit system when LREA is used.

The proposed algorithm provides better numerical convergence for all the systems considered in the study. Currently, LR method is a widely accepted algorithm in real power system scheduling which is considered as a benchmark for comparing the results. It is shown in the paper that LREA provides better solutions.

#### Competitive environment

Similar to the cooperative environment, the feasibility of the proposed algorithms is demonstrated in competitive environment with different size of power systems containing 10–500 units. The same ten unit system is used in these studies also. Forecasted loads, spot prices, and reserve prices for the system are given in Table 2. In these studies, reserve prices are fixed at five times the spot prices, and probability that reserve is called and generated is kept at 0.05. Simulations were carried out and \$113134.12 profit is achieved by the proposed algorithm. It is higher than that from [22]. The corresponding power settings of each unit are given in Table 5.

Lagrangian relaxation and genetic algorithm are also used for generation scheduling of the same systems and used as benchmark algorithms. The best results obtained by the algorithm are tabulated in Table 6 for other systems containing different number of units.

The proposed LREA algorithm provides better numerical convergence for all the systems considered in the simulation, and provides better solutions.

LR for unit commitment problem provides a fast solution but it may suffer from numerical convergence and solution quality and Evolutionary Algorithm (EA) methods find the better solution but they need more computational time. In addition, LR decomposition procedure is dependent on the initial estimates of the Lagrangian multipliers and on the method used to update the multipliers. Currently the techniques used for estimating the Lagrangian multipliers rely on a sub-gradient algorithm or heuristics. Incorporating evolutionary algorithm into LR method to update the Lagrangian multipliers improves the performance of LR method in solving the unit commitment problem. As the result, it provides better numerical results obtained in a reasonable computational time.

#### Conclusion

This paper proposes a hybrid algorithm to solve the short-term generation scheduling problem with operational constraints in a traditional and a restructured power systems using Lagrangian relaxation and evolutionary algorithm. In the restructured power system, spot prices and reserve prices in the market are important parameters while solving the profit based unit commitment problem. Using the proposed LREA approach, the generation companies can maximize their profit and schedule the generating units accordingly. The results achieved are quite encouraging and indicate the feasibility of the proposed technique to deal with large unit commitment problems in a deregulated environment. Furthermore, the proposed algorithm can find the most economical scheduling plan for traditional power systems according to the results. Using the proposed approach, the system can minimize the operating cost of generators substantially. According to the numerical results, LREA seems to be the best algorithm among the algorithms discussed in this paper for short-term generation scheduling problems in both cooperative and competitive energy environments.

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