Exponentially Stable Lyapunov Function Based Controller for Flyback CCM Converter

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Abstract—In this paper, we propose a Lyapunov function based controller for the flyback converter operating in the continuous conduction mode (CCM). The controller consists of a duty-ratio feedforward control unit and a Lyapunov function based feedback control unit. The duty-ratio feedforward signal is used to reduce the burden from the Lyapunov feedback controller. The control system guarantees global exponential stability of the closed-loop system and hence offers fast transient response under large-signal perturbations. In constructing the controller, we use the average model of the flyback CCM converter taking the parasitic components into account. Numerical simulations confirmed its superior performance and experimental tests validated the proposed control approach.

Index Terms—Lyapunov stability, nonlinear controller, nonlinear systems, dynamic response, parasitic components

I. INTRODUCTION

The dc/dc converters based on flyback topology have been widely used in the industrial areas such as electronic devices, telecommunication equipments, and automotive systems due to its design simplicity, cost effectiveness, and isolation characteristics [1–3]. The flyback dc/dc converter can be designed to operate either in the discontinuous conduction mode (DCM) or in the continuous conduction mode (CCM). The flyback converter operating in CCM is more efficient with lower current stress and easier to design a filter over those operating in DCM, and hence there have been considerable interests in using the flyback CCM converter for industrial applications [4–7]. However, it suffers from the nonminimum phase behavior because its transfer function from the control duty to the output voltage has the right-half-plane zero [8]. The output voltage control of the flyback CCM converter is a challenging issue.

With regard to control design of the flyback CCM converter, the linear control design techniques have been popular particularly when the converter operates around a fixed operating point. To design a linear controller systematically, we usually need a linearized model of the converter which cannot reflect the complete dynamic behavior of the flyback CCM converter. Due to this reason, it is usually difficult to stabilize the control system with the linear controller particularly under large perturbations from the operating point.

To overcome the problem, a number of researchers have proposed to use the nonlinear controllers to control the inherently nonlinear converter. It is developed directly from the nonlinear converter model and hence does not need any linearization process. A number of nonlinear control techniques such as backstepping control techniques, sliding mode control (SMC) techniques and model predictive control (MPC) techniques have been developed for the flyback CCM converter. The backstepping control is the stepwise control design technique based on the Lyapunov function and can regulate the output voltage of the converter by stabilizing the inductor current and guarantees closed-loop system stability [9]. But it is sensitive to parameter variations and suffers from slow transient response due to the inherent singularity problem. The SMC is a robust control technique that can cope with parameter variations, external disturbances, and unmodeled dynamics with fast transient performance. In its early stage, the hysteresis-modulation (HM)-based SMC has been widely used to regulate the output voltage of converters [10]. However, the HM-based SMC generates a bang-bang type control input, and so the controlled system generally suffers from chattering and significant switching-frequency variation when the input voltage and the output load are varied. These complicate the design of the input and output filters and deteriorate regulation properties of the converters. Thus, SMC that operates at constant switching frequency has been developed by employing pulse-width-modulation (PWM) instead of HM. An equivalent control is the low-frequency continuous switching action that will produce a trajectory that is of near equivalence to an ideal sliding mode trajectory [11]. This equivalent control can solve the drawbacks of the HM-based SMC such as the chattering and variable switching frequency problems. However, it would come with the steady-state error in the output voltage due to imperfect switching control action [12–14]. MPC offers a solution for trajectory optimization [15] but, because MPC calculates its control input from the dynamic model of the system, discrepancies between the actual system and the dynamic model such as uncertainties and external disturbances can deteriorate the performance of MPC. Moreover, due to large amount of computation in
the optimization process, MPC is not suitable for the high frequency switching converter.

The Lyapunov function based control strategy, which usually offers a solution that guarantees global asymptotic stability under large perturbations from the operating point. Motivated from the excellent properties of the Lyapunov function based control approach, this controller has been successfully implemented in the buck-boost converter [16], boost converter [17], single- and three-phase shunt active power filters [18, 19], single-phase grid-connected PV inverters [20], single-phase uninterruptible power system (UPS) inverters [21], single-phase grid-connected voltage source inverter (VSI) [22], and switched reluctance motor drive system [23]. In the Lyapunov function based control scheme, a control signal such that the total energy of the system is continuously dissipated has been designed. The state of the closed-loop system then converges to the equilibrium point.

In this paper, we propose a Lyapunov function based feedback controller to be used for the CCM flyback converter. The flyback CCM converter suffers from the nonminimum phase behavior due to the existence of right-half-plane zero in its transfer function. So, it shows poor transient response under the large signal perturbation. But, thanks to global exponential stable characteristic of Lyapunov function based controller, the transient response becomes significantly improved. To reduce the burden from the Lyapunov function based controller, we supplement it with the duty-ratio feedforward controller [24–26]. We take the parasitic components into account to develop the dynamic model of the flyback CCM converter, and use it to construct the proposed controller; it regulates the reference output voltage more accurately. We performed numerical simulations to demonstrate the superior performance and conducted experimental tests to validate the proposed control scheme.

This paper is organized as follows. We introduce the modeling of flyback converter with parasitic components in Section II. We present controller design and stability analysis in Section III. To validate the proposed controller, we show the simulation and experimental results in Section IV. Finally, we draw conclusions in Section V.

II. SYSTEM CONFIGURATION

A. Modeling of the flyback CCM converter with parasitic components

Due to the difficulties and complexities of the modeling and controller design procedure, the parasitic components are usually ignored. The resultant ideal/lossless modeling significantly simplifies model development, and thereby contributes to understanding of the main features of the switching converter. However, the effects of parasitic components are important for improving the model accuracy and study of the dynamic performance. Use of a model that includes the parasitic components can improve steady-state performance as well as transient performance. Therefore, we take parasitic components into account to construct the dynamic model of the flyback CCM converter.

Fig. 1 describes the circuit diagram of the flyback converter with parasitic components. It consists of switch $S_i$, transformer $T$ with turns ratio $n = N_p/N_s$, magnetizing inductor $L_m$, diode $D$, and output capacitor $C$. The parasitic components are presented as leakage inductor $L_k$, the parasitic resistance of the switch $R_{sw}$, the direct current resistance of the magnetizing inductance $R_{m}$, the parasitic resistance of the diode $R_d$, and the voltage drop of the diode $V_d$; $R_{sw}$ represents the resistance between the source and the drain of the switch during the switch on state [27, 28].

The flyback converter is operated as follows: when the switch is turned on, the magnetizing inductance of the transformer accumulates the electric energy from the input power source; when the switch is turned off, the stored energy then transfers to the secondary winding when the voltage direction is reversed; when the diode is turned on, the magnetizing energy transfers to the output capacitor and the load. Fig. 2 describes the waveforms of the primary current, secondary current, and magnetizing current. The magnetizing current increases when the switch is turned on. Otherwise, the magnetizing current decreases. Since the flyback converter is designed to operate in CCM, the magnetizing current never goes to zero during the switching period.

By applying Kirchhoff’s voltage and current laws, the state-space equations corresponding to the switch turned-on or turned-off cases can be expressed in the following equations, respectively.

![Circuit diagram of the flyback CCM converter with parasitic components](image1)

![Current waveforms of the flyback CCM converter](image2)
Switch turned-on:
\[
(L_m + L_k) \frac{di_{Lm}}{dt} = -(R_{Lm} + R_{sw})i_{Lm} + V_i, \tag{1}
\]
\[
\frac{di_{lk}}{dt} = \frac{di_{Lm}}{dt}, \tag{2}
\]
\[
C \frac{dv_o}{dt} = -\frac{1}{R_v} v_o, \tag{3}
\]

Switch turned-off:
\[
L_m \frac{di_{Lm}}{dt} = -(R_{Lm} + n^2 R_d)i_{Lm} - n v_o - n V_d, \tag{4}
\]
\[
\frac{di_{lk}}{dt} = 0, \tag{5}
\]
\[
C \frac{dv_o}{dt} = n i_{Lm} - \frac{1}{R_v} v_o, \tag{6}
\]

where \( i_{Lm} \) is the current through the magnetizing inductor, \( i_{lk} \) is the current through the leakage inductor, \( V_i \) is the input voltage, and \( v_o \) is the output voltage.

Combining (1) through (6) using the averaging technique, we obtained the average model that can be expressed as
\[
\frac{di_{Lm}}{dt} = -\left\{ \frac{(R_{Lm} + R_{sw})u_1}{L_m + L_k} + \frac{1}{L_m} (n^2 R_d + R_{Lm})(1-u) \right\} i_{Lm} - \frac{n(1-u)}{L_m} v_o + \frac{V_i u}{L_m + L_k} - \frac{n V_d (1-u)}{L_m}, \tag{7}
\]
\[
\frac{di_{lk}}{dt} = -\frac{(R_{Lm} + R_{sw})u_2}{L_m + L_k} i_{Lm} + \frac{V_i u}{L_m + L_k}, \tag{8}
\]
\[
\frac{dv_o}{dt} = \frac{n(1-u)}{C} i_{Lm} - \frac{v_o}{C R_v}, \tag{9}
\]

where \( u \) is the control duty (0 ≤ u ≤ 1) and \( i_{Lm}, i_{lk} \) and \( v_o \) are average values of \( i_{Lm}, i_{lk} \) and \( v_o \) during the switching period.

### III. Controller Design and Stability Analysis

The control objective of the flyback CCM converter is to make the output voltage track the reference signal as closely as possible. Conventional proportional-integral (PI) control scheme is usually designed using the linearized model of the converter and has been widely used to stabilize the converter. But it shows poor transistion response under load signal perturbations because the small signal model at each operating point is not accurate. To accomplish fast dynamic response and derive global exponential stability even under large signal perturbations, we propose to use a Lyapunov function based controller. To reduce the burden from the Lyapunov function based controller, we supplement it with the duty-ratio feedforward signal. In this section, we first develop the duty-ratio feedforward control part and use it to derive the error dynamics. The resulting error dynamic model is then used to develop the Lyapunov function based controller.

#### A. Duty-ratio feedforward controller

For subsequent development, we define the reference magnetizing inductor current as \( I_{Lm} \) and the reference output voltage as \( V_o \) and assume that the flyback CCM converter operates in the steady-state. Substituting \( I_{Lm} \) and \( V_o \) into \( i_{Lm} \) and \( v_o \), and setting \( \frac{di_{Lm}}{dt} = 0 \) and \( \frac{dv_o}{dt} = 0 \) in the average model, we have
\[
I_{Lm} = \frac{-n(1-u) V_o}{L_m} + \frac{V_i u}{L_m + L_k} - \frac{n V_d (1-u) V_o}{L_m}, \tag{10}
\]
\[
I_{Lk} = \frac{R_{Lm} + R_{sw} V_o}{L_m + L_k}, \tag{11}
\]
\[
V_o = n R_o (1-u) F I_{Lm}, \tag{12}
\]

Multiplying both sides of (10) by \( \frac{(R_{Lm} + R_{sw}) u_{ff}}{L_m + L_k} + \frac{1}{L_m} (n^2 R_d + R_{Lm})(1-u) \) and rearranging (10), we have
\[
\left\{ \frac{(R_{Lm} + R_{sw}) u_{ff}}{L_m + L_k} + \frac{1}{L_m} (n^2 R_d + R_{Lm})(1-u) \right\} I_{Lm} + \frac{n (1-u)}{L_m} V_o - \frac{V_i u}{L_m + L_k} = 0. \tag{13}
\]

Substituting (12) into (13), we then obtain
\[
\left\{ \frac{(R_{Lm} + R_{sw}) u_{ff}}{L_m + L_k} + \frac{1}{L_m} (n^2 R_d + R_{Lm})(1-u) \right\} I_{Lm} + \frac{n (1-u) V_o}{L_m + L_k} - \frac{V_i u_{ff}}{L_m + L_k} + \frac{n V_d (1-u) V_o}{L_m + L_k} = 0. \tag{14}
\]

which in turn can be arranged as
\[
\left\{ -L_m (R_{Lm} + R_{sw})(1-u) + R_{Lm} + R_{sw} \right\} I_{Lm} + \frac{n (1-u) V_o}{L_m + L_k} + \frac{V_i u}{L_m + L_k} + \frac{n V_d (1-u) V_o}{L_m + L_k} = 0. \tag{15}
\]

Multiplying both sides of (15) by \( n R_o (1-u) \) results in
\[
n R_o \left\{ n (L_m + L_k) V_o + L_m V_i + n (L_m + L_k) V_d u_{ff} \right\} - \frac{n R_o L_m V_i + L_m (R_{Lm} + R_{sw}) - (L_m + L_k)}{(n^2 R_d + R_{Lm}) V_o} u_{ff} + L_m (R_{Lm} + R_{sw}) V_o = 0 \tag{16}
\]

where \( u_{ff} = 1 - u \). \( u_{ff} \) can then be calculated from (16) as
\[
u_{ff} = \frac{G_1 \pm G_2}{G_3}, \tag{17}
\]

where
\[
G_1 = \frac{n R_o L_m V_i + L_m (R_{Lm} + R_{sw}) - (L_m + L_k)}{(n^2 R_d + R_{Lm}) V_o}, \tag{18}
\]
\[
G_2 = \sqrt{\left\{ n R_o L_m V_i + L_m (R_{Lm} + R_{sw}) - (L_m + L_k) \right\}^2 - 4 n R_o L_m (R_{Lm} + R_{sw}) (n L_m + L_k) V_o}, \tag{19}
\]
\[
G_3 = 2 n R_o \left\{ n (L_m + L_k) V_o + L_m V_i + n (L_m + L_k) V_d \right\}. \tag{20}
\]

Between the two values of \( u_{ff} \), the one with the minus sign in the numerator makes the reference magnetizing current to
become too large. Thus, we use the following duty-ratio
\[
  u_{ff} = 1 - \frac{G_1 - G_2}{G_3},
\]
(21)

$u_{ff}$ itself does not directly determine the output voltage of flyback CCM converter. It is used as a feedforward input so that the output voltage of the converter tracks the reference output voltage closely. In other words, the feedforward controller reduces the burden from the feedback controller, which therefore does not require a high gain in the feedback controller.

### B. Lyapunov function based controller

To construct a Lyapunov function based controller, we first apply the duty-ratio feedback signal $u_{ff}$ to the converter and derive the error dynamics. We first define the magnetizing inductor current error as $e_i = \frac{1}{L_m} - I_{lm}$, the output capacitor error as $e_v = \bar{v}_o - V_o$, and the feedback control term as $u_{fb} = u - u_{ff}$. Substituting $e_i$, $e_v$, and $u_{fb}$ into (7) and (9), we obtain
\[
  \dot{e}_i = \left\{- \frac{R_{lm} + R_{sw}}{L_m + L_k} u_{ff} + \frac{1}{L_m} (n^2 R_d + R_{lm})(1-u_{ff}) \right\} e_i - \frac{n(1-u_{ff})}{L_m} e_v + \left( \frac{R_{lm} + R_{sw}}{L_m + L_k} + \frac{n^2 R_d}{L_m} \right) e_i - \left( \frac{R_{lm} + R_{sw}}{L_m + L_k} + \frac{n^2 R_d}{L_m} \right) e_v + \frac{R_{lm}}{L_m} I_{lm} + \frac{n}{L_m} V_o + \frac{n V_d}{L_m} \right\} u_{fb},
\]
(22)
\[
  \dot{e}_v = \frac{n(1-u_{ff})}{C} e_i - \frac{1}{C R_o} e_v - \frac{n I_{lm}}{C} u_{fb},
\]
(23)
The error dynamics (22) and (23) can be compactly described as
\[
  \dot{e} = A e + (B e + b) u_{fb},
\]
(24)
where
\[
  e = \begin{bmatrix} e_i \\ e_v \end{bmatrix},
\]
(25)
\[
  A = \begin{bmatrix}
    -\frac{R_{lm} + R_{sw}}{L_m + L_k} u_{ff} + \frac{n^2 R_d + R_{lm}}{L_m} (1-u_{ff}) & \frac{n(1-u_{ff})}{L_m} \\
    \frac{n I_{lm}}{C} & -\frac{1}{C R_o}
  \end{bmatrix},
\]
(26)
\[
  B = \begin{bmatrix}
    \frac{R_{lm} + R_{sw}}{L_m + L_k} + \frac{n^2 R_d + R_{lm}}{L_m} & \frac{n}{L_m} \\
    \frac{n}{L_m} V_o + \frac{n V_d}{L_m} & 0
  \end{bmatrix},
\]
(27)
\[
  b = \begin{bmatrix}
    \frac{R_{lm} + R_{sw}}{L_m + L_k} + \frac{n^2 R_d + R_{lm}}{L_m} I_{lm} \\
    -\frac{n}{L_m} I_{lm}
  \end{bmatrix}.
\]
(28)

According to the Lyapunov stability theorem [29], the closed-loop system becomes exponentially stable if the Lyapunov function $V(x)$ satisfies $e_i \|x\|^a \leq V(x) \leq c_2 e_i \|x\|^a$ and $\dot{V}(x) \leq -c_3 e_i \|x\|^a$ where $0 < c_1 < c_2$, $0 < c_3$ and $a$ can be any positive constant.

If we choose the Lyapunov function candidate as the energy stored in the magnetizing inductor and output capacitor:
\[
  V(e) = \frac{1}{2} L_m e_i^2 + \frac{1}{2} C e_v^2 = \frac{1}{2} e^T P e,
\]
(29)
where $P = \text{diag}[L_m, C]$, then it holds that
\[
  \frac{1}{2} \min(L_m, C) \|e\|^2 = \frac{1}{2} \lambda_{\min}(P) \|e\|^2 \leq V(e) \leq \frac{1}{2} \lambda_{\max}(P) \|e\|^2.
\]
(30)
Computing its derivative with respect to time, we obtain
\[
  \dot{V}(e) = \frac{1}{2} e^T P e + \frac{1}{2} e^T \dot{P} e + \frac{1}{2} e^T \dot{e}_i \|e\|^2
\]
\[
  = \frac{1}{2} \{ A e + (B e + b) u_{fb} \}^T \{ A e + (B e + b) u_{fb} \}
\]
\[
  = \frac{1}{2} e^T (A^T P + P A) e + \frac{1}{2} u_{fb} e^T (B^T P + P B) e
\]
\[
  + u_{fb} b^T P e
\]
\[
  = -e^T Q e + u_{fb} e^T Ke + u_{fb} b^T Pe,
\]
(31)
where
\[
  Q = \frac{1}{2} (A^T P + P A)
\]
\[
  = \begin{bmatrix}
    \frac{L_m (R_{lm} + R_{sw})}{L_m + L_k} u_{ff} + (n^2 R_d + R_{lm})(1-u_{ff}) & 0 \\
    0 & 0
  \end{bmatrix} > 0,
\]
(32)
\[
  K = \frac{1}{2} (B^T P + P B)
\]
\[
  = \begin{bmatrix}
    \frac{L_m (R_{lm} + R_{sw})}{L_m + L_k} u_{ff} + (n^2 R_d + R_{lm})(1-u_{ff}) & 0 \\
    0 & 0
  \end{bmatrix}.
\]
(33)

When we choose a Lyapunov function based feedback controller as
\[
  u_{fb} = -\alpha (e^T Ke + b^T Pe),
\]
(34)
where $\alpha > 0$, the time derivative of the Lyapunov function becomes
\[
  \dot{V}(e) \leq -e^T Q e
\]
\[
  \leq -\lambda_{\min}(Q) \|e\|^2
\]
\[
  \leq \min \left( \frac{L_m (R_{lm} + R_{sw})}{L_m + L_k} u_{ff} + (n^2 R_d + R_{lm})(1-u_{ff}) + \frac{1}{R_o} \right) \|e\|^2
\]
\[
  < 0.
\]
(35)
From (30) and (35), we can conclude from the Lyapunov stability theorem that $e_i \to 0$ and $e_v \to 0$ exponentially fast as $t \to \infty$. In fact, combining (30) and (35), we obtain
\[
  \dot{V}(e) \leq -\frac{2 \lambda_{\min}(Q)}{\lambda_{\max}(P)} V(e),
\]
(36)
and solving (36) gives
\[ V(e(t)) \leq V(e(t_0)) \exp \left\{ -\frac{2\lambda_{\min}(Q)}{\lambda_{\max}(P)} (t - t_0) \right\}, \quad (37) \]
for all \( t \geq t_0 \). Then the error trajectory is bounded as
\[
\|e\| \leq \left\{ \frac{2V(e(t_0))}{\lambda_{\min}(P)} \right\}^{1/2} \leq \left[ \frac{2V(e(t_0))}{\lambda_{\min}(P)} \right]^{1/2} \lambda_{\max}(P) \left\{ ||e(t_0)||^2 \exp \left\{ -\frac{2\lambda_{\min}(Q)}{\lambda_{\max}(P)} (t - t_0) \right\} \right\}^{1/2} = \left\{ \lambda_{\max}(P) \right\}^{1/2} \left\{ \frac{2V(e(t_0))}{\lambda_{\min}(P)} \right\}^{1/2} \lambda_{\min}(P) \left\{ ||e(t_0)||^2 \exp \left\{ -\frac{2\lambda_{\min}(Q)}{\lambda_{\max}(P)} (t - t_0) \right\} \right\}^{1/2}. \quad (38)\]

Consequently, the error trajectories converge to zero exponentially fast with the rate of \(-\frac{2\lambda_{\min}(Q)}{\lambda_{\max}(P)}\).

The complete control input is then determined from (21) and (34) as
\[
u = u_{ff} + u_{fb} = 1 - \frac{G_1 - G_2}{G_3^2} - \alpha(e^T Ke + b^T Pe). \quad (39)\]

The proposed control scheme (Fig. 3) consists of two components: the duty-ratio feedforward control term that helps the output voltage to track the reference voltage, and the Lyapunov function based feedback control term that drives the closed-loop system to become exponentially stable.

In order to design the proposed controller, we need to measure the current through the magnetizing inductor. The magnetizing inductor current is not usually measurable directly. But, as shown in Fig. 2, \( i_L \) can be expressed as the sum of \( i_{pri} \) and \( i_{sec}/n \), and we can then determine \( i_L \) by measuring \( i_{pri} \) and \( i_{sec} \). Another approach is to adopt the state observer that can estimate the magnetizing current thereby removing the need to use the two current sensors. In this paper, we focus on demonstrating the feasibility of the proposed control scheme and simply assume that we use two current sensors instead of developing a state observer technique.

IV. SIMULATION AND EXPERIMENTAL RESULTS

In order to examine the transient response of the closed-loop system under variations of the reference output voltage,
input voltage, and output load, we first conducted simulation tests using the computer simulation software PowerSim V9.0 (PSIM). Next, we carried out experimental tests using the prototype of the flyback CCM converter controlled by TMS320F28377D digital signal processor (Fig. 4). The major parameters of the flyback CCM converter are listed in Table I and the total system configuration is shown in Fig. 5.

To verify the performance of the proposed controller, we have compared the proposed controller with the voltage-mode controller, the current-mode controller, and the sliding mode controller. Type-II control scheme is adopted to construct the voltage mode controller for the flyback CCM converter.

\[ G_{vc}(s) = \frac{k_{vc}}{s + w_{vc.z}}, \]  

(40)

where \( k_{vc} \) is the integral gain of the voltage-mode controller, \( w_{vc.z} \) is the zero of the voltage-mode controller, and \( w_{vc.z} \) is the pole of the voltage-mode controller. Parameters of the voltage-mode controller were first set to satisfy the design criterion: the phase margin 55.9° and the closed-loop bandwidth 3.76 kHz; \( k_{vc}, w_{vc.z}, \) and \( w_{vc.p} \) were set to 0.002, 3.5 and 300. We tested the control performance of the voltage-mode controller by changing the parameters around the preset value. Since we conducted the parameter optimization process, we obtained considerably improved result when \( k_{vc}, w_{vc.z}, \) and \( w_{vc.p} \) were set to 0.004, 4.5 and 180 in the simulation test. As we selected the control parameters in experiment, a similar tendency was observed in the control performance of simulation and experiment. We obtained the experimental result when \( k_{vc}, w_{vc.z}, \) and \( w_{vc.p} \) were set to 0.001, 7.1, and 210.

The current-mode controller consists of the inner-loop current controller and the outer-loop voltage controller. The controller structure in the outer-loop voltage loop is the same as that in voltage-mode controller.

\[ G_{cc}(s) = \frac{k_{cc}}{s + w_{cc.z}}, \]  

(41)

where \( k_{cc} \) is the integral gain of the current-mode controller, \( w_{cc.z} \) is the zero of the current-mode controller, and \( w_{cc.z} \) is the pole of the current-mode controller. Parameters of the current-mode controller were first set to satisfy the design criterion: the phase margin 77.4° and the closed-loop bandwidth 3.9 kHz; \( k_{cc}, w_{cc.z}, \) and \( w_{cc.p} \) were set to 0.1, 2.5 and 150. Using the same manner as in voltage-mode controller, we finally obtained \( k_{cc}, w_{cc.z}, \) and \( w_{cc.p} \) as 0.3, 2.3 and 140 in the simulation and 0.6, 5.0 and 170 in experiment. These algorithms are built on TMS320F28377D microcontroller, which offers advantages of lower sensitivity to parameter variation, programmability and possibilities to improve performance using more advanced control schemes [30].

PWM based sliding mode controller is adopted to construct the sliding mode controller as in [33]. The sliding surface has been selected as

\[ S = \lambda_1 e_t + \lambda_2 e_o + \lambda_3 \int e_t dt + \lambda_4 \int e_o dt, \]  

(42)

where \( e_t \) and \( e_o \) are current and voltage errors, and \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) are positive constants.

Since the system has been linearized around the equilibrium point, root-locus technique is first used to analyze the effect of the parameters [31–33]. For parameter \( \lambda_1 \) and \( \lambda_2 \), the eigenvalues go close to real axis but do not go to the right-half plane as \( \lambda_1 \) and \( \lambda_2 \) increase respectively. For parameter \( \lambda_3 \), the eigenvalues go to minus infinity as \( \lambda_3 \) increases. For parameter \( \lambda_4 \), the eigenvalues lie in the open left-half complex plane with complex roots as \( \lambda_4 \) increases. Thus, with any positive parameter values, all of the eigenvalues lie in the open left-half complex plane and the linearized system is stable. Next, the parameters of the sliding mode controller were first determined to satisfy the design criterion: the phase margin 93.9° and the bandwidth 8.38 kHz; \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) were set to 0.001, 0.025, 3.2 and 6.8 respectively. Then, we tested the control performance of the sliding mode controller by changing the parameters around the preset value. Since we conducted the parameter optimization process, we obtained \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) as 0.005, 0.039, 2.0 and 6.5 in the simulation and 0.017, 0.04, 4.5 and 7.0 in the experiment.

For the proposed controller, we tested the performance of the controller by increasing the value of \( \alpha \). Since we did the parameter tuning process of proposed controller, we obtained the controller gain \( \alpha = 0.004 \) in the simulation and \( \alpha = 0.002 \) in the experiment.

A. Simulation Results

Simulation tests were performed to examine the transient response of the closed-loop system under variations of the reference output voltage, input voltage, and output load. The reference output voltage changed from 21 V to 15 V under the input voltage \( V_i = 12 \) V and the resistive load \( R_o = 20 \) Ω. The reference output voltage \( V_o \) was set to 21 V during the first 0.1 s, to 15 V during the next 0.1 s, and to 21 V during the rest 0.1 s. The input voltage changed from 10 V to 15 V under the reference output voltage \( V_o = 18 \) V and the resistive load \( R_o = 20 \) Ω. The input voltage \( V_i \) was set to 10 V during the first 0.1 s, to 15 V during the next 0.1 s, and to 10 V during the rest 0.1 s. The output load changed from 10 Ω to 20 Ω under the reference output voltage \( V_o = 18 \) V and the input voltage \( V_i = 12 \) V. The output load \( R_o \) was set to 20 Ω during the first 0.1 s, to 10 Ω during the next 0.1 s, to 20 Ω during the rest 0.1 s.

Fig. 6 shows the waveforms of the actual and reference output voltages under reference output voltage variations. As shown in Fig. 6(a), the actual output voltage tracks the reference output voltage rather slowly when the voltage-mode controller is used. Fig. 6(b) shows the waveforms of the actual and reference output voltages when the current-mode controller is used. Comparing the waveforms of Fig. 6(b) with those of Fig. 6(a), we see that the actual output voltage tracks the reference output voltage faster but its settling time is still too long. Fig. 6(c) shows the waveforms of the actual and reference output voltages when the sliding mode controller is used. The actual output voltage shows large overshoot and some oscillation before settling down to the reference output.
voltage but its settling time is faster than that of Fig. 6(a) and Fig. 6(b). Fig. 6(d) shows the waveforms of the actual and reference output voltages when the proposed controller is applied. The output voltage indeed tracks the reference output voltage with significantly reduced overshoot and settling time. A similar tendency is observed in the output voltage under input voltage variations. Fig. 7 shows the waveforms of the actual and reference output voltages under output load variations. Here, the voltage-mode controller works better than the current-mode controller. It is due to the fact that the output impedance of the closed-loop system under voltage-mode control is smaller than that under current-mode control. When the sliding mode controller is used, the output voltage shows less peak behavior (Fig. 8(c)). When the proposed controller is used, the output voltage keeps its value well even under load variations (Fig. 8(d)).

To demonstrate the performance quality of the proposed controller, we examined the rise time and settling time of the transient responses under reference output voltage variations, the percentage of overshoot and settling time under input voltage variations, and the percentage of overshoot and settling time under output load variations (Tables II, III, and IV). The proposed controller exhibits the fastest and most stable step response among different types of controllers.

### Table II
<table>
<thead>
<tr>
<th>Controller</th>
<th>Rise time</th>
<th>Setting time</th>
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</thead>
<tbody>
<tr>
<td>Voltage-mode controller</td>
<td>9.7 ms</td>
<td>12.9 ms</td>
</tr>
<tr>
<td>Current-mode controller</td>
<td>6.2 ms</td>
<td>9.0 ms</td>
</tr>
<tr>
<td>Sliding mode controller</td>
<td>1.2 ms</td>
<td>3.1 ms</td>
</tr>
<tr>
<td>Proposed controller</td>
<td>0.8 ms</td>
<td>2.0 ms</td>
</tr>
</tbody>
</table>

### Table III
<table>
<thead>
<tr>
<th>Controller</th>
<th>Percentage of overshoot (PO)</th>
<th>Setting time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage-mode controller</td>
<td>30.8 %</td>
<td>10.2 ms</td>
</tr>
<tr>
<td>Current-mode controller</td>
<td>18.7 %</td>
<td>8.8 ms</td>
</tr>
<tr>
<td>Sliding mode controller</td>
<td>9.8 %</td>
<td>3.2 ms</td>
</tr>
<tr>
<td>Proposed controller</td>
<td>6.4 %</td>
<td>1.6 ms</td>
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### Table IV
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<tr>
<th>Controller</th>
<th>Percentage of overshoot (PO)</th>
<th>Setting time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage-mode controller</td>
<td>18.8 %</td>
<td>5.2 ms</td>
</tr>
<tr>
<td>Current-mode controller</td>
<td>20.9 %</td>
<td>7.9 ms</td>
</tr>
<tr>
<td>Sliding mode controller</td>
<td>10.6 %</td>
<td>2.0 ms</td>
</tr>
<tr>
<td>Proposed controller</td>
<td>5.4 %</td>
<td>0.7 ms</td>
</tr>
</tbody>
</table>

**B. Experimental Results**

The experiment was also performed to test the transient response of the closed-loop system under variations of the input voltage.
Fig. 7. Simulation waveforms of the flyback CCM converter under input voltage variations. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.

Fig. 8. Simulation waveforms of the flyback CCM converter under output load variations. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.
reference output voltage, input voltage, and output load. The reference output voltage is 2.5-Hz square-wave voltage that jumps between 21 V to 15 V. The input voltage is 2.5-Hz square-wave voltage that jumps between 10 V to 15 V. The output load is 2.5-Hz square-wave that jumps between 10 Ω to 20 Ω.

Fig. 9 shows the waveforms of the input/output voltages and inductor current under reference output voltage variations. As shown in Fig. 9(a), the output voltage tracks the reference output voltage rather slowly when the voltage-mode controller is used. Fig. 9(b) shows the waveforms of the input/output voltages and inductor current when the current-mode controller is used. Comparing the waveforms of Fig. 9(b) with those of Fig. 9(a), we see that the output voltage tracks the reference output voltage faster but its settling time is still too long. Even though we increase the controller gain to improve the transient performance, we cannot noticeably improve the transient response. Fig. 9(c) shows the waveforms of the input/output voltages and inductor current when the sliding mode controller is used. The output voltage tracks the reference output voltage with an acceptably short settling time. Fig. 9(d) shows the waveforms of the input/output voltages and inductor current when the proposed controller is applied. Thanks to the global exponential stability property of the proposed controller, the output voltage is able to track the reference output voltage well even during the transient period. Fig. 10 and Fig. 11 show the waveforms of the input/output voltages and inductor current under input voltage variations and output load variations. In agreement with simulation results, a similar tendency is observed in the waveforms. Fig. 12 shows the waveforms of the input/output voltages during the converter startup. During the startup transient, the proposed controller quickly tracks the reference output voltage without overshoot.

We also examined the rise time and settling time of the transient responses under reference output voltage variations, the percentage of overshoot and settling time under input voltage variations, and the percentage of overshoot and settling time under output load variations (Tables V, VI, and VII). The proposed controller also shows the fastest and most stable step response among different types of controllers.

### Table V

<table>
<thead>
<tr>
<th>Controller</th>
<th>Rise time</th>
<th>Settling time</th>
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</thead>
<tbody>
<tr>
<td>Voltage-mode</td>
<td>10.7 ms</td>
<td>13.7 ms</td>
</tr>
<tr>
<td>Current-mode</td>
<td>7.4 ms</td>
<td>10.2 ms</td>
</tr>
<tr>
<td>Sliding mode</td>
<td>0.9 ms</td>
<td>1.7 ms</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.8 ms</td>
<td>1.2 ms</td>
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### Table VI

<table>
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<th>Controller</th>
<th>Percentage of overshoot (PO)</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage-mode</td>
<td>37.3 %</td>
<td>19.1 ms</td>
</tr>
<tr>
<td>Current-mode</td>
<td>26.7 %</td>
<td>13.4 ms</td>
</tr>
<tr>
<td>Sliding mode</td>
<td>12.2 %</td>
<td>7.6 ms</td>
</tr>
<tr>
<td>Proposed</td>
<td>9.1 %</td>
<td>3.5 ms</td>
</tr>
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</table>

### Table VII

<table>
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<tr>
<th>Controller</th>
<th>Percentage of overshoot (PO)</th>
<th>Settling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage-mode</td>
<td>11.3 %</td>
<td>11.5 ms</td>
</tr>
<tr>
<td>Current-mode</td>
<td>12.7 %</td>
<td>13.6 ms</td>
</tr>
<tr>
<td>Sliding mode</td>
<td>7.3 %</td>
<td>4.3 ms</td>
</tr>
<tr>
<td>Proposed</td>
<td>5.2 %</td>
<td>1.1 ms</td>
</tr>
</tbody>
</table>

To reduce the burden from the Lyapunov function based controller, we supplement it with the duty-ratio in the feed-forward loop. To achieve accurate control, we derive the average model of the flyback CCM converter taking the parasitic components into account and use the derived model to develop the controller; it regulates the reference output voltage more accurately. Numerical simulations confirmed its superior performance and experimental tests validated the proposed control approach.

### V. Conclusion

In this paper, we proposed a Lyapunov function based controller for the CCM flyback converter with design simplicity, cost effectiveness and isolation characteristics in mind. The Lyapunov function based controller guarantees global exponential stability of the closed-loop system and it provides fast transient response under the large-signal perturbations.

### References


Fig. 9. Experimental waveforms of the flyback CCM converter under reference output voltage variations. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.
Fig. 10. Experimental waveforms of the flyback CCM converter under input voltage variations. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.


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Fig. 11. Experimental waveforms of the flyback CCM converter under output load variations. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.

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Fig. 12. Experimental waveforms of the flyback CCM converter during system startup. (a) when the voltage-mode controller is used. (b) when the current-mode controller is used. (c) when the sliding mode controller is used. (d) when the proposed controller is used.