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Hydroelectric unit commitment for power plants composed of distinct groups of generating units

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ABSTRACT

This paper proposes a new solution method for the unit commitment problem of hydro generation units for power systems in which there is prevalence of hydroelectric participation. Attention is focused on hydro power plants composed of distinct groups of generating units, each of which exhibiting different performance characteristics. Considering the forecasted load curves provided by short term operation planning studies, an optimization problem is formulated to determine how many and which generating units should be dispatched in each hour and at which power plant so as to meet load, operational and energy target constraints. The latter are established by medium term operation planning studies. The objective function to be minimized comprises distinct components related to loss of efficiency in the use of water resources, as well as unit startup and shutdown costs. The difficulties posed by the discontinuous nature of the loss of performance function, which actually comprises a collection of distinct curves, are dealt with by aggregating them into a continuous surface obtained via best fitting methods. This artifice allows the application of efficient mixed integer programming algorithms to solve the hydro unit commitment problem, and is seen as a contribution of this paper. The performance of such optimization model is assessed via its application to real power plants of the Brazilian interconnected power system. The results show that the proposed operation policy promotes the optimum use of water resources, leading to the minimum depletion of reservoirs as compared to all other unit combinations.

1. Introduction

Unit Commitment (UC) is a key function in power system short-term operations planning which aims at determining which generating units are the most cost-effective to be dispatched, in order to supply the forecasted demand and spinning reserve requirements over time horizons ranging from one day to one week. Several technical constraints and economical factors such as generating unit minimum up and down times, start-up costs and shutdown costs, etc., should be taken into account in the solution, making the UC of thermal units one of the most complex problems in power system operation [1]. Once UC determines the generator schedule for a given time horizon, Economic Dispatch is able to assume a fixed set of available generating units in order to compute the generation levels for any point of the demand curve pertaining to that horizon.

A variety of approaches to solve the UC problem can be found in the literature, including heuristic methods, computational intelligence techniques and theoretically denser optimization methods. Detailed overviews are presented in [2] and more recently in the comprehensive survey [3], which also encompasses uncertain aspects, as well as in [4], whose focus is on deterministic approaches for the hydro UC problem. Certainly due to its immediate economical impacts, the thermal UC problem has received much more attention in the literature than its hydroelectric counterpart. This fact reflects itself in the operation practices adopted by the industry, particularly in those countries in which hydro power is prevalent. Nonetheless, the hydro UC problem has been the object of interest of several research groups worldwide, and distinct approaches have been proposed to address the problem, such as computational intelligence methods [5–7], Dynamic Programming [8], Lagrangian relaxation [9–12], Mixed Integer Linear Programming (MILP) [13,14], and Mixed Integer Nonlinear Programming (MINLP) [15].

In [9,10], a sequential quadratic programming approach and Lagrangian relaxation techniques are employed to solve the hydro-thermal UC problem, and bundle methods are also used to update

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the Lagrange multipliers. Along the same line, the authors of [11] focus attention on the hydro generation optimal dispatch sub-problem, making use of augmented Lagrangian techniques. The resulting method is then applied to determine the optimal dispatch of two hydro plants of the Brazilian power system.

Other references address the hydro UC problem from the point of view of minimizing the Loss-of-Performance (LoP) in the hydroelectric generation process due to factors such as tailrace elevation, penstock head losses and turbine-generator efficiency variation. In [12], a pre-dispatch model aimed at minimizing both transmission power losses and hydro generation loss-of-performance due to the above factors is developed. The optimization problem is solved through a combination of Lagrange relaxation and heuristic techniques. The same concepts regarding generation LoP are applied in [16] to a single hydro power plant. The number of generating unit start-ups/shut-downs also plays an important role in this context. A method based on Dynamic Programming which contemplates the above aspects is presented in [16] and applied to a large hydro plant comprising 18 identical units. The results reported in that paper show that the number of dispatched units significantly affects the overall power plant efficiency. In [17], the same performance criteria are extended for application to a Brazilian hydroelectric system formed by several power plants. The resulting optimization problem is solved via a heuristic procedure based on Lagrangian relaxation. The comparison of the proposed strategy with actual system operation during a typical day reveals that the former yields considerable savings of energy due to improvements on system overall efficiency.

In this paper, we consider the *short term* hydroelectric UC problem with a time horizon of typically one to a few days, so that reservoir inflows are assumed known and a deterministic approach can be used. Unlike the cases dealt with in [13–15], large reservoirs typical of the Brazilian power system are considered, so that the influence of reservoir water level on the power plant output (the so-called "head effect") can be neglected [9,12,16]. No pumping storage operation is taken into account. Also unlike [13–15], we assume a *centralized* schedule of *multiple reservoirs* located in a given water basin, conducted by a single operator. Moreover, the operation environment is not price-based, that is, the aim is not the maximization of producer revenues. Instead, the main objective is to maximize the efficient use of water resources. For that purpose, it is assumed that an energy target in MWh determined in mid-term studies for each power plant, in addition to the forecasted power demand, must be satisfied.

We adopt the hydroelectric UC problem formulation described in [16,17], but introduce an alternative solution method that avoids the use of heuristic techniques or an straightforward application of Dynamic Programming without problem decomposition, which could turn out to be a computationally inefficient strategy for problems comprising a large number of generating units. As in the above references, the proposed approach is aimed at minimizing the loss-of-performance in the hydro generation process due to: (i) tailrace elevation, (ii) penstock head losses, and (iii) turbine-generator efficiency variation. A challenge associated with this approach is that the LoP function for a given hydro plant considering the above factors is not a continuous, analytical function. Instead, it takes the form of a collection of mappings, one for each combination of generating units in operation. To cope with that, we propose a new strategy based on the aggregation of the various LoP mappings into a single objective function. This is accomplished by defining a continuous surface through best fitting methods that approximately contains all individual LoP curves. In addition, the number of units in operation is treated as an additional (integer) optimization variable. The objective function is also extended to account for the cost of generating unit start-ups and shut-downs. This results in a Mixed Integer Nonlinear Programming (MINLP) problem, which is solved by appropriate computational tools. Finally, we generalize the problem formulation in order to take into account the real case of power plants comprising distinct groups of identical generating units. Although this problem has been addressed before using other approaches [9], to the best of our knowledge the methodology based on minimizing loss-of-performance components has not been previously applied to such general case.

This paper is organized as follows. In Section 2 the basic concepts related to the modeling of hydroelectric power plants are reviewed. The characterization and mathematical formulation of the various factors considered in the definition of the LoP function are discussed in Section 3, which also deals with the aggregation of LoP components for distinct numbers of generating units in operation. Section 4 presents the overall formulation of the hydroelectric UC problem. Results of several case studies solved through the proposed approach are presented and discussed in Section 6. Finally, the concluding remarks are listed in Section 7.

2. Mathematical model of hydro units

Consider a hydro power plant i composed of J_i generating units, and that n_i of them are in operation, $n_i \leq J_i$. The electric power $p_{i,j}$ (in MW) generated by hydro unit j is a function of the corresponding turbine water discharge rate $q_{i,j}$ (in m^3/s), the net water head $h_{i,j}$ (in m) and the unit combined turbine-generator efficiency $\eta_{i,j}$, and is given by

$$p_{i,j} = K\eta_{i,j}h_{i,j}q_{i,j} \qquad (2.1)$$

where K is a constant equal to 0.00981 MJ/m^4. Eq. (2.1) is often referred to as the hydro unit *production function*. It is more complex than one could think at first glance, due to the fact that the involved variables are interdependent. To begin with, turbine-generator efficiency is actually a nonlinear function of $h_{i,j}$ and $p_{i,j}$ (or, equivalently, q_{ij}). Since in general there is no analytical form for the hill curve, it is in practice approximated by a second order polynomial of the form:

$$\eta_{i,j}(h_{i,j}, p_{i,j}) = \gamma_{0,i,j} + \gamma_{1,i,j}h_{i,j} + \gamma_{2,i,j}p_{i,j} + \gamma_{3,i,j}h_{i,j}p_{i,j} + \gamma_{4,i,j}h_{i,j}^2 + \gamma_{5,i,j}p_{i,j}^2 \qquad (2.2)$$

where coefficients $\gamma_{k,i,j}$, k = 0, 1, ..., 5 are previously determined through "best fitting" (multivariable linear regression) methods by using points obtained from the actual hill diagram provided by the manufacturer. It should be noticed that, for a given net water head, as the power output increases the efficiency also increases, until a maximum efficiency value is attained. Further increases on the power output lead to declining efficiency values. A similar reasoning can be applied when varying water head for a fixed power output. In other words, there is a point of maximum efficiency $\eta_{ij}^{\max}$ (the so-called "design point") for which the hydro unit reaches the optimal energy conversion performance.

The water head in Eq. (2.1) depends upon the forebay water level, b_i, and the tailrace level, r_i. The difference $(b_i - r_i)$ is the *gross* water head, but pressure losses due to friction of the water in the penstock have also to be accounted for. The latter are usually expressed in meters and considered as a quadratic function of the unit discharge:

$$h_{ij}^{loss} = \beta_j q_{i,j}^2 \qquad (2.3)$$

where β_j (in s^2/m^5) is a constant whose value depends on the characteristics of the particular installation. The above considerations lead to the following expression for the net water head:

$$h_{i,j} = b_i - r_i - h_{ij}^{loss} \tag{2.4}$$

Forebay elevation is in turn a function of the reservoir water storage. In short term operations planning studies encompassing only a few days' horizon, variations on the forebay level tend to be negligible, especially when dealing with large reservoirs [16].

The same does not apply, however, to the tailrace level, which may significantly vary with total hydro plant water release D_i even for a short term horizon. For hydro plant i, D_i comprises both power plant discharge and spillage rates, denoted as Q_i and S_i, respectively, so that

$$D_i = Q_i + S_i \tag{2.5}$$

Although included in the above formulation, spillage is usually undesirable and only justifiable under very particular operating conditions dictated by exogenous factors. Therefore, it will not be considered for the remaining of this paper. That is to say, it is assumed that the plant water release basically depends upon the corresponding discharge rate.

No analytical relationship between tailrace level r_i and total water release D_i is available. Hence, it has to be determined from experimental data. The model used in the Brazilian system operations planning studies is a fourth-order polynomial approximation of the form:

$$r_i(D_i) = \kappa_0 + \kappa_1 D_i + \kappa_2 D_i^2 + \kappa_3 D_i^3 + \kappa_4 D_i^4 \tag{2.6}$$

where coefficients κ_k, $k = 0, 1, \ldots, 4$ are determined by best fitting techniques.

Since (i) the total water release is ultimately a function of the discharge of each unit, and (ii) the number of units in operation may vary according to loading and dispatch conditions, it is important to clearly establish the relationship between Q_i and q_{ij}. When power plant i comprises a single group of identical generating units and assuming that identical units are dispatched so as to produce the same power output, we have

$$Q_i(n_i) = n_i q_i \tag{2.7}$$

3. Main loss-of-performance components

From the developments in Section 2, the total power output of power plant i can be written as

$$P_i = K\eta_i(n_i)h_i(n_i)Q_i(n_i) \tag{3.1}$$

where η_i is the plant overall efficiency and h_i is the plant net head. Assuming all units in operation are identical and, as mentioned before, that identical units are dispatched with equal power outputs, the plant overall efficiency is equal to the individual unit efficiency value. As discussed in the previous section, unit efficiency is a nonlinear function of net water head and turbine discharge, often referred to as *hill diagram*. It is represented either as a collection of contour lines [10] or as three-dimensional plots (see [12,16] for examples of real hydro unit 3-D hill diagrams). Similarly to Q_i (see Eq. (2.7)), both η_i and h_i are also functions of n_i, the number of units in operation, since varying n_i affects the unit discharge rates q_{ij} and consequently the unit power outputs P_{ij}. Eqs. (2.2), (2.4)–(2.6) then clearly show that η_i and h_i also change with n_i.

Therefore, given an expected electric power demand to be met by the power plant, it becomes relevant to determine which number of units in operation produces the "best" performance in some sense. In this paper, we follow Refs. [12,16,17] and define such best condition as the one that minimally deviates from ideal performance index values. In other words, we search for the unit commitment that minimizes a set of LoP components, as discussed in the sequel.

The three main LoP components considered in this paper are [12]: (a) deviation of turbine efficiency values with respect to the point of maximum efficiency, denoted by LoP_η; (b) reduction of net water head due to elevation of tailrace level, LoP_r; and (c) reduction of net water head due the penstock water friction, LoP_f. Each one of those components are discussed and mathematically modeled in the following subsections.

3.1. Loss-of-performance due to deviations from maximum turbine efficiency

As briefly discussed in Section 2, there is a particular combination of net water head-output power values which yield the maximum power plant efficiency value $\eta_i^{\max}$. Typical unit hill diagrams [12] indicate that operating a generating unit at a point that significantly deviates from such "design point" leads to considerable loss-of-performance in the energy conversion process.

For a given number of generation units in service, n_i, this LoP component can be determined from Eq. (3.1) and from the maximum efficiency value $\eta_i^{\max}$ of power plant i, obtained from the corresponding hill diagram. Accordingly,

$$(LoP_\eta)_i = K(\eta_i^{\max} - \eta_i(n_i))h_i(n_i)Q_i(n_i) \tag{3.2}$$

Eq. (3.2) is directly applicable for the case of a power plant composed of a single group of identical generating units, but can be easily generalized for the case of two or more groups of units.

3.2. Loss-of-performance due to tailrace elevation

An increase on the power plant total water discharge Q_i, either by opening the wicket gates of units already in service or by putting more units into operation, produces an increase of power plant output. However, either action also increases the tailrace level r_i, leading in turn to a reduction of the net water head, as given by (2.4). The final result is an *effective* power plant output somewhat smaller than the value that would result without tailrace elevation. From Eq. (3.1), one can easily conclude that the corresponding loss-of-performance component can be written as

$$(LoP_r)_i = K\eta_i(h_i^{ref} - h_i(n_i))Q_i(n_i) \tag{3.3}$$

where h_i^{ref} is a reference value for the net water head, properly chosen to prevent the occurrence of negative values for $(LoP_r)_i$ over all expected operating conditions. In [16], this value is defined as the one corresponding to the minimum plant water discharge with n_i units in operation. Using this definition and (2.4) under the hypothesis of constant forebay elevation, a reference value r_i^{ref} corresponding to h_i^{ref} can be defined for the tailrace level, which leads to an alternative form of (3.3):

$$(LoP_r)_i = K\eta_i(r_i(n_i) - r_i^{ref})Q_i(n_i) \tag{3.4}$$

3.3. Loss-of-performance due to frictional head losses

The third LoP component is related to the loss of water pressure due to water friction in the unit penstocks, and is sometimes referred to as *penstock head losses or hydraulic losses*. As discussed in Section 2, such loss of pressure in meters is usually approximated by a quadratic function of the discharge rate. Similarly to tailrace elevation, its effect is also to reduce the net water head, as given by Eq. (2.4), and can be expressed in *MW* by an equation similar to (3.3).

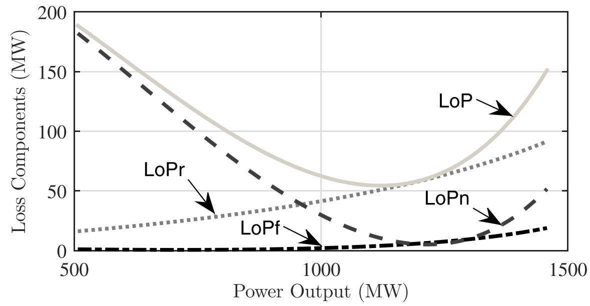

Fig. 1. Aggregated LoP and LoP components for Marimbondo hydro power plant with all eight generating units in operation.

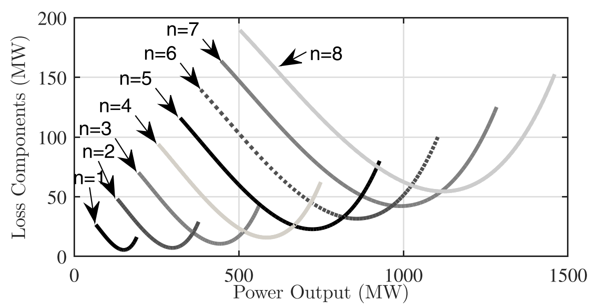

Fig. 2. Aggregated LoP curves for Marimbondo power plant with varying number of units in operation.

Also as in the previous case, a particular discharge value chosen to fulfill the same requirements as before can be adopted. From that reference discharge value and the assumption that n_i generation units are in operation, a head loss value $h_i^{loss,ref}$ can be computed from Eq. (2.3). Using $h_i^{loss,ref}$ as the frictional head loss reference, the corresponding loss-of-performance component in MW is given by

$$(LoP_f)_i = K\eta_i(h_i^{loss}(n_i) - h_i^{loss,ref})Q_i(n_i) \tag{3.5}$$

3.4. Typical generation loss-of-performance curves

Assuming that all technical data for a given hydro power plant are available, including the polynomial approximations for the hill curves (Eq. (2.2)), tailrace elevation (Eq. (2.6)), and frictional head losses (Eq. (2.3)), curves for the LoP components as functions of the output power can be generated by gradually increasing unit discharge rates from its minimum up to its maximum value, under the assumption of constant forebay level previously discussed. The Marimbondo hydro power plant, which is part of the Brazilian power system, is used in this paper to illustrate the concept of LoP curves. Such power plant is composed of eight identical 180 MW units, so that its maximum power output is 1440 MW. Considering that all units are in operation, Fig. 1 shows the curves for three LoP components, as well as for the aggregated LoP, as the plant is dispatched from its minimum up to its maximum power output under the described conditions. The aggregated LoP is defined simply as

$$LoP = LoP_\eta + LoP_r + LoP_f \tag{3.6}$$

It is also possible to determine LoP × output power as the number of units in operation varies. This is shown in Fig. 2. For a given value of the plant power output, one can promptly see from Fig. 2 that there may be distinct unit combinations for the same dispatch, but there is an optimal number of units in operation for which LoP is minimum. Since this implies a minimum depletion of water volume in the reservoir, as shown in the following sections, LoP minimization is a fundamental aspect to be considered in short term hydro schedule studies.

Figs. 1 and 2 illustrate the fact that the total loss-of-performance index for a given hydro power plant is ultimately a function of two variables, namely, the number of generating units actually in operation and the power output of each unit. Sticking to the assumption that identical units are equally dispatched, we can express the LoP of power plant i at a given time t as a function $LoP_i : \mathbb{R}^{n_i} \times \mathbb{N} \to \mathbb{R}$ defined as

$$LoP_i(t) = f_i(p_i^t, n_i^t) \tag{3.7}$$

where p_i^t is the power output of each generating unit of power plant i in operation at time t, n_i^t is the number of units of hydro plant i in operation at time t. As illustrated in Fig. 2, f_i is a "family" of nonlinear mappings. Each of those mappings corresponds to a particular value of n_i^t and is determined as given by Eq. (3.6) considering that n_i^t generating units are in operation.

4. Mathematical formulation of the hydro UC problem considering distinct unit groups

4.1. Hydro stations with multiple unit groups

In this paper, the hydroelectric UC approach based on minimizing LoP is generalized so as to consider power plants comprising distinct groups of identical generating units, the units of each group exhibiting performance characteristics which differ from each other. As a consequence, the notation employed in the previous section has to be expanded in order to accommodate the presence of such groups.

First of all, indices i, ℓ, and j consistently refer to power plants, groups of units and generating units, respectively. Let us assume that the power system under study is composed by I hydro power plants, and that power plant i contains L_i distinct groups of units. Group ℓ of plant i is in turn composed by a total of $J_{i\ell}$ units, of which $n_{i\ell}^t$ are actually in operation at time interval t. Let $p_{i,\ell}^t$ be the power output generated by each of those units at time t (recall that all identical units are dispatched with the same power output). Accordingly, Eqs. (2.7) and (3.1) can be easily generalized to furnish plant total discharge, Q_i^t, and plant total power output, P_i^t, at time t, for the case of multiple groups of identical units.

4.2. Mathematical formulation of the hydro UC problem

The hydro UC problem is formulated in this paper as an optimization problem whose main objective is to minimize the total loss-of-performance of the generation system during a given time horizon. This time horizon is discretized into T intervals, each of them of duration h^t, $t = 1, \ldots, T$. In the same optimization process, unit startup and shutdown costs are simultaneously taken into account.

The optimal unit commitment solution must ensure that the output power of the committed power plants meets the forecasted power demand at each interval of the time horizon. In addition, it is assumed that there is an energy target in MWh assigned to each power plant that must be also fulfilled. Such a target is a function of the water volume available for power generation throughout the whole time horizon, and is previously determined by midterm operations planning studies [18]. The remaining constraints refer to the relationships between the number of operating units within each group and the statuses of such units as determined by the optimization process; enforcement of lower and upper limits on unit output powers; and the binary nature of the variables representing unit statuses. The status of generating unit j of group ℓ pertaining to power plant i at time interval t is denoted by $u^t_{i,\ell,j}$, and is equal to 1 if the unit is on and zero if it is off.

The hydroelectric unit commitment problem is stated in (4.1). The cost function to be minimized comprises two types of terms: the first one represents the aggregated LoP costs of all power system hydro plants, whereas the second term takes into consideration unit startup and shutdown costs. Parameters $c^L_{i,\ell}$ (in \$/MW) and $c^S_{i,\ell}$ (in \$) stand for the estimated LoP unitary cost and the startup/shutdown cost, respectively, both assigned to a generating unit of group ℓ pertaining to plant i. Values for the latter type of cost parameters can be estimated from recommendations of previous studies [19] and unit rated powers. Function $f_{i\ell}(p^t_{i,\ell}, n^t_{i\ell})$ is simply a generalization of (3.7) for the case of plants comprising multiple groups of units. The products of terms depending on unit status in the objective function of (4.1) account for the number of unit startups and shutdowns at consecutive time intervals.

In the first constraint of the optimization problem (4.1), P^t_D is the system load at time interval t, while the left-hand side of the equation represents the sum of the power generated by all power plants (transmission losses are neglected). M_i in the second constraint denotes the energy target in MWh for power plant i, for the whole time horizon. It defines intertemporal constraints at power plant level, that must be met by the outputs of operating units pertaining to distinct generating groups. The third constraint in (4.1) establishes the relationship between the binary variables indicating the active units pertaining to a given group at interval t and the number of the group's generating units in operation at the same interval. The remaining constraints enforce either physical limits on unit output powers and number of units or variable types. It is important to notice that the main optimization variables of Problem (4.1) are generating unit power outputs and number of units in operation at each time interval, what is in compliance with the discussion in Section 3.4 related to Eq. (3.7).

$$\min_{p^t_{i,\ell}, n^t_{i\ell}} \sum_{t=1}^{T} h^t \left\{ \sum_{i=1}^{I} \sum_{\ell=1}^{L_i} \left[c^L_{i,\ell} \times f_{i\ell}(p^t_{i,\ell}, n^t_{i\ell}) + \sum_{j=1}^{J_{i\ell}} c^S_{i,\ell} \times ((u^t_{i,\ell,j}(1-u^{t-1}_{i,\ell,j}) + u^{t-1}_{i,\ell,j}(1-u^t_{i,\ell,j})) \right] \right\}$$

subject to :

$$\begin{aligned}
&\sum_{i=1}^{I}\sum_{\ell=1}^{L_i} n^t_{i\ell} p^t_{i,\ell} = P^t_D, \quad t = 1, \ldots, T \\
&\sum_{t=1}^{T} h^t \sum_{\ell=1}^{L_i} n^t_{i\ell} p^t_{i,\ell} \le M_i, \quad i = 1, \ldots, I \\
&\left.\begin{aligned} &\sum_{j=1}^{J_{i\ell}} u^t_{i,\ell,j} - n^t_{i\ell} = 0 \\ &0 \le n^t_{i\ell} \le J_{i\ell} \end{aligned}\right\} \quad \begin{cases} \ell = 1, \ldots, L_i; \\ i = 1, \ldots, I; \\ t = 1, \ldots, T. \end{cases} \\
&\underline{p}_{i,\ell} \le p_{i,\ell} \le \bar{p}_{i,\ell} \\
&u^t_{i,\ell,j} \in \{0, 1\}, \quad \forall i, j, \ell, t
\end{aligned} \tag{4.1}$$

5. Solution of the hydro UC problem

For real power systems, Problem (4.1) is a large and challenging optimization problem, since it is nonlinear, contains both continuous and integer variables, and includes time-coupling terms as constraints and also in its objective function. However, the main difficulty for its solution resides in the fact that the LoP component in the objective function is discontinuous and composed of multiple nonlinear segments, which precludes the application of analytical solution methods. An alternative is to resort to techniques based on directly computing objective function values, but since they depend on the varying number of generating units in operation, a considerable amount of decision making and branching instructions have to be repeatedly executed to obtain LoP values. Dynamic Programming (DP) is a valid candidate [16], usually combined with some sort of heuristic procedures to cope with the effects of the so-called "curse of dimensionality" usually associated with DP. Another possibility is to apply metaheuristic methods, such as Evolutionary Algorithms, ant colony methods, etc. [5,6]. However, depending on the number of generating units involved, the large computational effort required by such approaches may turn out to be a significant hurdle for accommodate hydroelectric UC solutions in short term planning studies.

In this paper, we propose a novel approach to cope with the difficulties posed by the discontinuous nature of the LoP objective function components. It consists in replacing the discontinuous LoP component for group ℓ of power plant i by an equivalent function that takes the form of a surface $S_{i\ell} \in \mathbb{R}^3$, mathematically described by an analytical, polynomial expression.

The procedure to obtain surface $S_{i\ell}$ from LoP curves such as those of Fig. 2 can be seen as composed of two steps. First of all, a third dimension is added to Fig. 2 as an extra horizontal axis on which the number of generating units in operation is represented. This allows that the family of curves in Fig. 2 be "unstacked", that is, separated one from another onto distinct planes, each of which corresponding to a particular $n_{i\ell}$ value.

The second step consists in temporarily assuming that the number of units in operation is a continuous variable denoted by $x_{i\ell}$. Then, a surface described by a polynomial expression on $p_{i,\ell}$ and $x_{i\ell}$ is adjusted to the various LoP curves. This can be accomplished by means of surface fitting methods based on least-squares techniques. The degree of the fitted polynomial can be selected in order to produce good adherence to the individual LoP curves. Our experience points out that best results are obtained with forth-order polynomials.

Fig. 3 presents the LoP surface obtained from the curves for the Marimbondo power plant previously shown in Fig. 2. Traces in black correspond to the individual curves in that figure, while the fitted surface is represented in blue.

The definition of a LoP surface for each group of a given power plant allows that the hydro UC problem be restated as the Mixed Integer Nonlinear Problem (MINLP) given by (5.1). Notice that Problem (5.1) differs from Problem (4.1) in that: (a) the continuous and analytical LoP surface $S_{i\ell}$ replaces the discontinuous function $f_{i\ell}$ in the objective function; (b) variable $x^t_{i\ell}$ replaces $n^t_{i\ell}$, and (c) the integral nature of $x^t_{i\ell}$ is enforced by imposing an extra variable type constraint.

The solution for Problem (5.1) can be achieved via efficient MINLP solvers such as DICOPT (Discrete and Continuous Optimizer), devised to solve problems involving integer/binary and continuous variables [20], and available as part of the GAMS computational package. The algorithm underlying DICOPT relies on the relaxation of equality constraints through the imposition of penalties. Internally, DICOPT iteratively and alternately solves Mixed Integer Linear Programming and Nonlinear Programming subproblems until a solution is attained. The algorithm is able to deal with non-convexities, although it is not possible to ensure that the global optimum is always attained. In the current application, however, surface $S_{i\ell}$ tends to behave as a convex function within the usual operating range of the generating units, so that it is expected that

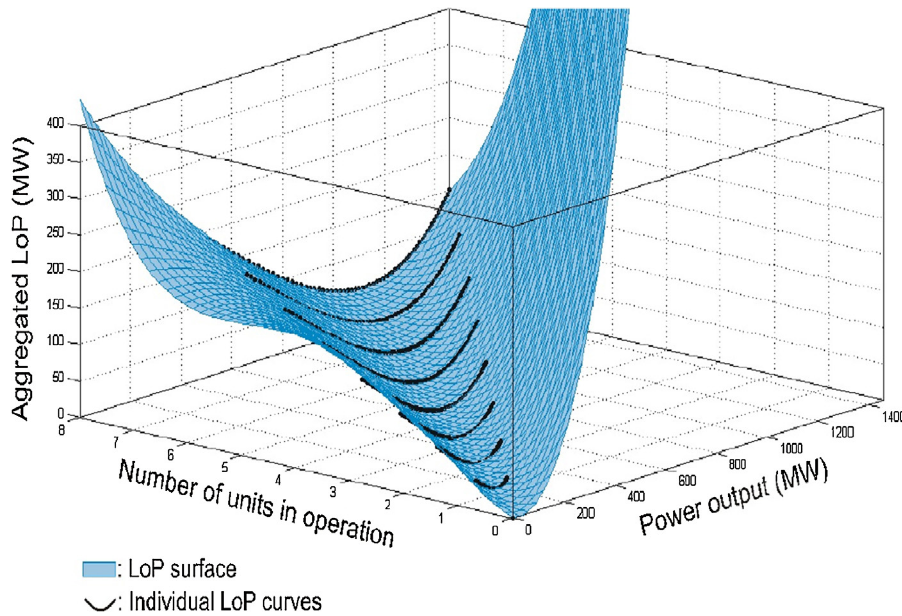

Fig. 3. Three-dimensional LoP surface for the Marimbondo hydro plant obtained through best fitting techniques applied to individual LoP curves. (For interpretation of the references to color in this sentence, the reader is referred to the web version of the article.)

the DICOPT provisions to handle non-convexities will suffice to provide the optimal solution.

$$
\begin{aligned}
&\min_{p^t_{i,\ell}, x^t_{i\ell}} \sum_{t=1}^{T} h^t \Bigg\{ \sum_{i=1}^{I} \sum_{\ell=1}^{L_i} \Big[c^L_{i,\ell} \times S_{i\ell}(p^t_{i,\ell}, x^t_{i\ell}) \\
&+ \sum_{j=1}^{J_{i\ell}} c^S_{i,\ell} \times ((u^t_{i,\ell,j}(1-u^{t-1}_{i,\ell,j}) + u^{t-1}_{i,\ell,j}(1-u^t_{i,\ell,j})) \Big] \Bigg\} \\
&\text{subject to :} \\
&\sum_{i=1}^{I} \sum_{\ell=1}^{L_i} x^t_{i\ell} p^t_{i,\ell} = P^t_D, \quad t = 1, \ldots, T \\
&\sum_{t=1}^{T} h^t \sum_{\ell=1}^{L_i} x^t_{i\ell} p^t_{i,\ell} \le M_i, \quad i = 1, \ldots, I \\
&\left.\begin{aligned} &\sum_{j=1}^{J_{i\ell}} u^t_{i,\ell,j} - x^t_{i\ell} = 0 \\ &0 \le x^t_{i\ell} \le J_{i\ell} \end{aligned}\right. \quad \left\{ \begin{aligned} &l\ell = 1, \ldots, L_i; \\ &i = 1, \ldots, I; \\ &t = 1, \ldots, T. \end{aligned}\right. \\
&\underline{p}_{i,\ell} \le p_{i,\ell} \le \bar{p}_{i,\ell} \\
&u^t_{i,\ell,j} \in \{0, 1\} \quad \text{and} \quad x^t_{i,\ell} \in \mathbb{N}, \quad \forall i, j, \ell, t
\end{aligned}
\tag{5.1}
$$

6. Results of case studies

In this section, results of hydro unit commitment studies conducted via the proposed method and involving two distinct test systems are presented. Two types of simulations are performed: (i) a study to confirm the optimality of the solution obtained with the proposed UC strategy and its impact on the total discharged water volume during a given time horizon, and (ii) a realistic simulation based on three existing hydro power plants located in the Southeastern region of the Brazilian power system, which include a total of 34 generating units. Results obtained for the two case studies are presented and discussed in the sequel.

In all simulated cases, our MINLP formulation (5.1) for the hydroelectric UC problem is solved using GAMS (General Algebraic Modeling System) computational package [21]. As mentioned in the previous section, the main GAMS solver employed for this problem is DICOPT [20], which in turn makes use of solvers CPLEX and CONOPT to obtain solutions for the mixed integer programming (MIP) and the nonlinear programming (NLP) subproblems, respectively.

6.1. Optimality analysis and impact on total discharged water volume

The objective of this subsection is to validate the proposed UC strategy through an enumerative procedure that lists all possible generating unit combinations that meet a load curve defined for a given time horizon. Clearly, such procedure requires that some practical limitations be imposed on the test system to be used, as well as on the load curve to be met, in order to prevent combinatorial explosion, that is, too large a number of feasible combinations to be examined, what would lead to an intractable problem.

Therefore, the test system to be used in this subsection is restricted to a single power plant, but which comprises two distinct groups of generating units. The first group is composed of three units whose minimum and maximum outputs are 55 MW and 230 MW, respectively, while the second group is formed by two units of 180 MW whose minimum output is 50 MW. As a consequence, the hydro station operating range goes from 50 MW up to 1050 MW. Although the power plant for this case study is hypothetical, its generating units correspond to real ones pertaining to existing hydro plants of the Brazilian power system: units of the first group have the same technical characteristics of units of Agua Vermelha hydro plant, while those in the second group replicate the characteristics of Marimbondo power plant units. Startup/shutdown costs are taken into account in the results that follows. The values of the corresponding coefficients $c^S_{i,\ell}$ have been estimated as suggested in Ref. [19] and are presented in Table 2 under the headings AV and M (for the units in the first and second groups, respectively). Of course, other values could have been used, what would affect the number of startup/shutdown unit switching operations in the final solution. For instance, large values assigned to $c^S_{i,\ell}$ would lead to a reduction on the number of switching operations, with the side effect of a corresponding decrease on the efficient use of the water resources.

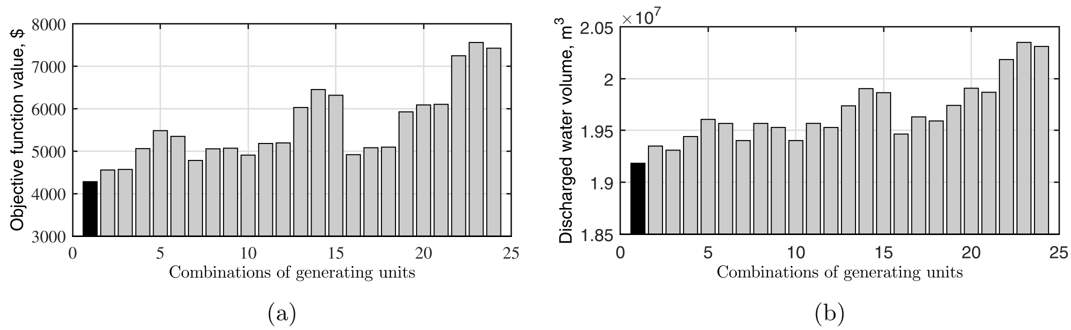

Fig. 4. (a) Objective function values and (b) discharged water volumes for several combinations of operating units.

Table 1
Load profile for case 1 and optimal UC.

Interval	1	2	3	4
Load (MW)	50	380	720	1050
Opt. solution	0–1	2–0	3–1	3–2

Loss-of-performance surfaces have been determined for each of the two generating groups by using the procedure described in Section 5. LoP curves similar to those in Fig. 2 had been previously obtained through the methodology presented in Section 3 from the technical data of Agua Vermelha and Marimbondo hydro stations.

This case study considers a 4-h operation planning horizon, divided into 1-h time intervals. Even for such a limited number of intervals and the assumption that at least one unit has to operate in each interval, the number of possible unit combinations amounts to 14, 641, whose processing through an enumerative procedure would demand a huge computational effort. Without loss of generality, we then make use of the load profile shown in Table 1, with which the number of *feasible* combinations is reduced to a manageable level. With such load curve, only 24 combinations are able to meet the demand without any unit violating its lower or upper generation limit.

The next step is to solve the optimization Problem (5.1) in order to identify which unit combination out of the 24 feasible ones provides the optimal UC solution. The optimal combination is shown in the last row of Table 1 as strings of the type $k_1 - k_2$, where k_ℓ is the operating units of group ℓ, ℓ=1, 2. Then, two performance indices are considered, for all feasible combinations: (a) the value of the cost function of Problem (5.1), and (b) the total discharged water volume during the whole 4-h time horizon. The results are plotted as bar charts in Fig. 4(a) and (b). Values provided by the optimal solution for Problem (5.1) are shown in black, while the calculated values corresponding to all other alternative combinations are represented in grey.

The results presented in Fig. 4(a) confirm that the unit combination determined by solving Problem (5.1) is in fact the one that exhibits the minimum objective function value. Although expected, those results validate the optimality of the unit combination provided by the proposed approach.

On the other hand, the results depicted in Fig. 4(b) are less expected, and convey an important meaning. They indicate that, out of all feasible solutions, the one that minimizes the power plant LoP over the whole time horizon tends to require the minimum amount of discharged water volume, and consequently produces the minimum reservoir water volume depletion. In other words, the proposed methodology of minimizing LoP components indirectly leads to the maximum water savings over the considered time horizon. Ultimately, the results of this case study confirm the superiority of the results produced by the proposed strategy over all other feasible solutions in terms of the efficiency of the overall energy conversion process.

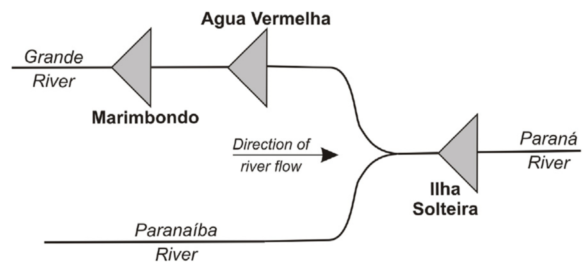

Fig. 5. Relative location of hydro stations in the river basin.

6.2. Application to a realistic hydro generation system

The realistic test system comprises three hydro stations of the Brazilian power system: *Marimbondo* power plant, located on the Grande river and formed by eight identical units of 180 MW; *Agua Vermelha* hydro station, also located on the same river and composed by six identical units of 232.7 MW; and *Ilha Solteira* hydro station, which is on the Paraná river and contains 20 generating units. The latter are divided into two groups: the first one, which we refer as *IS-I*, comprises four identical units of 176 MW, while group *IS-II* includes the remaining 16 units, whose rated powers vary between 170 and 174 MW. In this paper, however, we consider that all units in groups *IS-II* are identical, with rated power of 170 MW. Therefore, the installed capacity of the three hydro stations are 1440 MW, 1396.2 MW and 3424 MW, respectively. Their relative locations on the Paraná river basin is shown in Fig. 5. Although there are other hydro stations on the same basin, only those which compose the test system are represented in the figure. Altogether, 34 generating units are represented in this case study. The LoP surfaces for Marimbondo and Agua Vermelha hydro plants are shown in Figs. 3 and 6, whereas LoP surfaces for the two unit groups of Ilha Solteira power plant are depicted in Fig. 7. It should be mentioned that the domain of all LoP surfaces is defined by the individual curves shown in black in the figures. That is to say, the remaining parts of the surface beyond the black traces do not play any role in the optimization process.

The time horizon considered in the case study is a full day, divided into 24, 1-h intervals. The corresponding load curve is shown as the upper plot in Fig. 8(a), which exhibits peaks at hours 10 and 22. Startup/shutdown unit costs for units pertaining to each power plant group have been estimated as suggested in Ref. [19] and are presented in Table 2. The same table shows the assumed unit LoP costs within each generating group, and also energy targets M_i for each power plant during the whole 24-h time horizon. In practice, the latter are determined by longer term models that represent in detail the operation of the hydroelectric system [18].

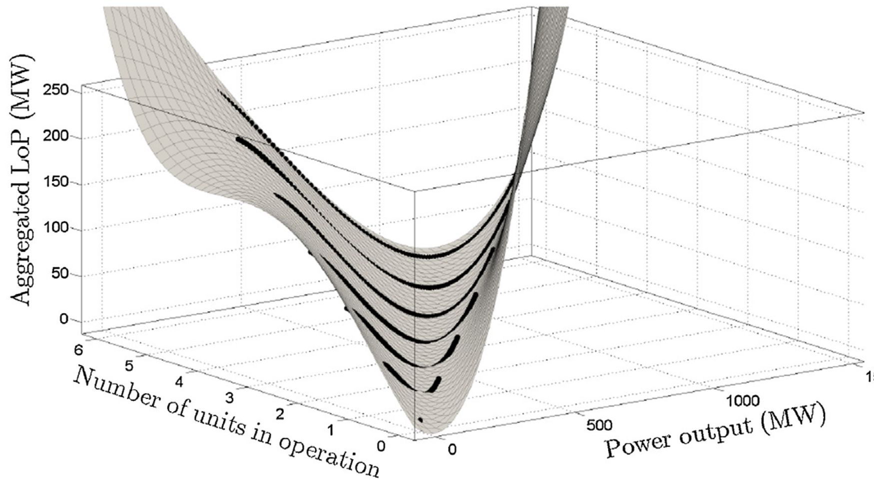

Fig. 6. LoP surface for the Agua Vermelha hydro plant.

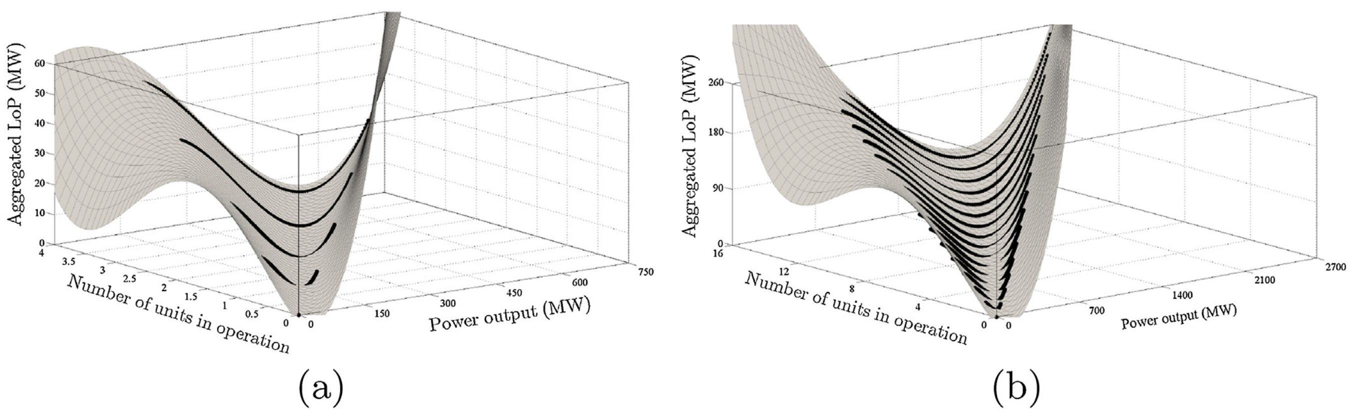

Fig. 7. LoP surfaces for the Ilha Solteira hydro plant: (a) group IS-I, comprising four generating units and (b) group IS-II, composed by 16 units.

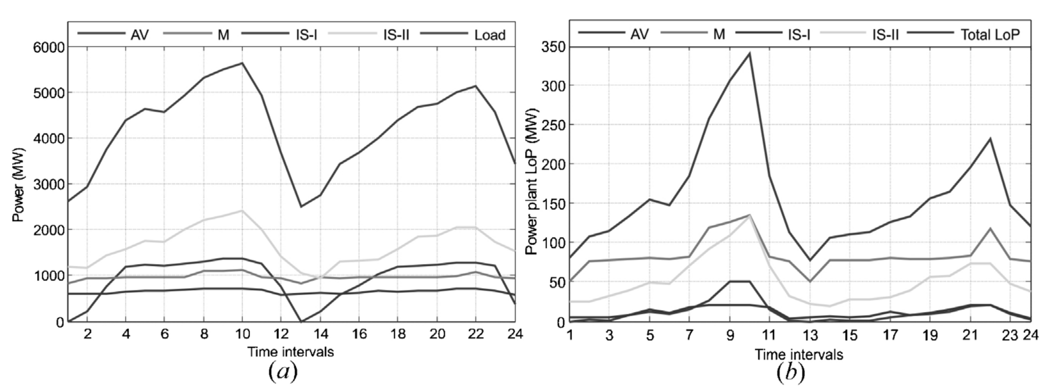

Fig. 8. (a) Load curve and output power curves and (b) total LoP curve and individual LoP curves for the four generating unit groups.

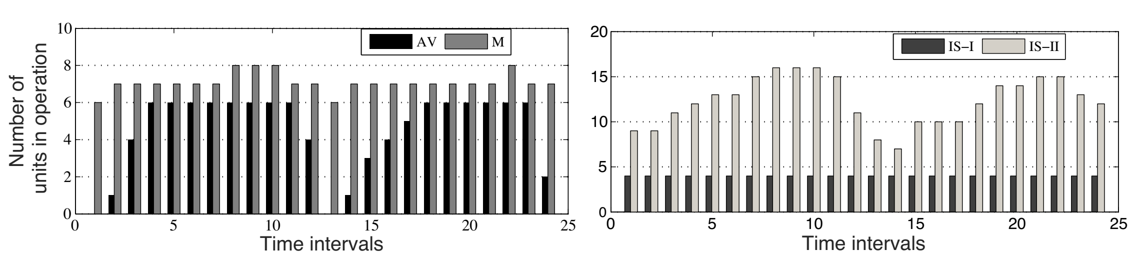

Fig. 9. UC results for the 4-hydro plant system. (a) Ilha Solteira IS-I and IS-II unit groups and (b) Agua Vermelha (AV) and Marimbondo (M) units.

Table 2
Operating costs and energy targets

Gen. unit group	St.up/sht.dn cost ($)	LoP cost ($/MW)	En. target (MWh)
AV	744	20	22,340
M	555	50	23,310
IS-I	528	20	55,430
IS-II	510	50	

In this paper, we assume a typical hydrologic scenario and, without loss of generality, establish energy target values in proportion to the power plant MW capacities. Although clearly an approximation, such criterion has produced results that, in relative terms, do not deviate too much from those obtained through more rigorous procedures [18]. Only one target value is provided for Ilha Solteira (*IS*) power plant, since energy targets apply to power stations rather than to unit groups.

Following the procedure described in Section 5, the first step toward the application of the proposed methodology to this test system is to determine individual LoP surfaces, similar to the one shown in Fig. 3, for each of the four generating unit groups. Space restrictions do not allow the presentation of the four LoP surfaces in this paper, but some remarks about them are in order at this point, as follows: (i) Among all generating groups, the range of LoP values in MW for group *IS-I* is narrower than the range of the others. (ii) That characteristic, together with the *IS-I* also moderate LoP unit cost given in Table 2, makes the *IS-I* LoP costs the lowest ones among all groups. (iii) By the same token, in the average the second lowest cost group is *AV*, followed by *IS-II*, and finally the single generating group *M* of the Marimbondo, which then exhibits the highest average LoP cost. Despite those observations, it should be clear that LoP costs are not the only factor that dictate unit commitment results, since the problem constraints, and in particular those related to energy targets, also play an important role in that respect.

The solution of Problem (5.1) for this test system using GAMS computational package takes six iterations of the MIP solver and seven iterations of the NLP solver until convergence is attained. Fig. 9 presents the UC results for the four generating group test system during a 24-h time horizon. For better visualization, the plots are grouped into two distinct charts, as described in the figure caption. The corresponding power outputs are depicted in Fig. 8(a). Finally, the evolution of the LoP values of the four generating groups along the 24-h period is shown in Fig. 8(b).

The UC results in Figs. 8(a) and 9(a) show that, at the optimal solution, all four units of group *IS-I* are dispatched at full capacity along the whole time horizon, whereas the commitments of units pertaining to group *IS-II* vary according to the load curve. Such an outcome is in agreement with the previous cost analysis above, and is made possible by the fact that both groups are subject to a single energy target constraint. This provides a certain degree of freedom within the whole set of *IS* hydro station units in the course of the optimization process, leading to a rather expected result.

The results for the other two groups are not so predictable, if one thinks only in terms of operating costs. As mentioned before, units of the Marimbondo hydro station tend to be the most costly ones. Nevertheless, this does not imply that at the solution they are less committed during the time horizon than, say, the *AV* units, whose operating costs are rather lower (see Fig. 9(b)). The explanation comes from the fact that each of the two power plants have its own energy target. As a consequence, the units of group *M* end up being committed and dispatched in all time intervals, as shown in Figs. 8(a) and 9(b), in order to fulfill its target. As a consequence, they eventually exhibit the highest LoP values throughout the period, according to Fig. 8(b). At the solution, the total LoP for the whole time horizon is 3803.6 MWh and the total discharged volume is 877.6 h m^3.

The optimization problem solved through GAMS comprises 2935 single equations, 39 blocks of equations, 2908 continuous and 936 discrete variables. The required CPU time to reach convergence on a Dual Core, 2.10 GHz, 64 bits, 3.0 GB RAM microcomputer has been 3 min and 24 s to solve a problem that altogether comprises 34 generating units. Even considering the evolution of computer technology over the last decades, this is certainly a competitive figure as compared with computing times reported in the literature to solve similar problems. In Ref. [12], for instance, which applies Lagrange relaxation and a heuristic post-processing stage to solve the hydro UC problem, CPU times of the same order of magnitude as the above are reported to solve a system composed of 12 generating units, that is, roughly one quarter of the size of our test system. In [9], a solution strategy based on Sequential Quadratic Programming and Lagrange relaxation demands 180 min to solve a larger problem, composed by 121 units (and thus about four times as large as the test system used in this paper).

7. Conclusions

The importance of the hydroelectric unit commitment problem lies mainly in the significant savings of water volume that its proper solution can promote, thereby contributing to reduce future operating costs. This paper address the problem from the point of view of minimizing the loss of performance (LoP) of groups of generating units, at the same time meeting demand constraints and power station energy targets established by longer term planning studies. The problem formulation considers the general case where a hydro station is composed by multiple groups of identical generating units. The discontinuous, multiple-segmented nature of the LoP curves of each group, usually seen as a deterrent factor for applying analytical optimization tools, is dealt with by determining LoP polynomial surfaces through best fitting methods. As a result, efficient mixed integer nonlinear algorithms can be successfully used to determine the most productive unit combination able to meet the above constraints. The optimality properties of the proposed approach has been validated by using a low dimension case study for which all possible unit combinations can be evaluated. In addition, its feasibility for application to real problems has been successfully demonstrated on a test system composed of three hydro station and 34 generating units that is part of the Brazilian power system.

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