

# A new approach to solve Economic Dispatch problem using a Hybrid ACO–ABC–HS optimization algorithm

Tanuj Sen, Hitesh Datt Mathur\*

*Department of Electrical and Electronics Engineering, BITS, Pilani, Rajasthan 333031, India*



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## ABSTRACT

This paper presents a newly developed hybrid optimization algorithm for solving the problem of Economic Dispatch (ED) for a multi-generator system. The hybrid algorithm named ACO–ABC–HS combines the framework of Ant Colony Optimization (ACO), Artificial Bee Colony (ABC) and Harmonic Search (HS) algorithms to find the optimized solution for the system. The ACO algorithm is used to find the initial solution set, the ABC algorithm is employed to test and improve each of the probable solutions provided by the ACO module, while the HS module is used to discard the inferior solutions from the solution set and replace them with better ones. The performance of this hybrid algorithm is compared with those of conventional ED solving techniques like Gradient Search as well as other evolutionary algorithms namely ABC, ACO, HS and Particle Swarm Optimization (PSO). Valve point loading, environmental emissions, line losses and ramping rate constraints have been included in the ED analysis to provide more practical results. The algorithm's performance is also tested for Multi-Area Economic Dispatch (MAED) with tie-line constraints. The results obtained clearly point out the superiority of the hybrid algorithm in finding out the optimum results, while satisfying the constraints of minimizing the generation costs, reducing the emissions as well as tie-line costs and transmission losses.

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## Introduction

Economic Dispatch (ED) is one of the central features of energy generation and distribution these days. With the demand for energy and the cost of fuel (for running the generators) both increasing monotonically, it has become increasingly important to find ways to reduce the cost of generation of energy. Economic Dispatch involves finding out the optimized load schedule for the generators in a power system in order to satisfy the entire power demand while incurring the minimum possible generation cost [1,2]. As a result, mathematical optimization techniques which can account for the characteristic cost function curves of generating units are required to be formulated and applied for finding the optimal load allocation.

The cost function of generators can be simply approximated by a second order non-linear function which is monotonically increasing in nature. However, such a simple cost function does not properly model the actual behavior of the generator which involves constraints like valve-point loading and multiple fuel usage [3].

Valve-point loading introduces ripples into the cost function causing multiple local minima to occur [4]. Prohibited zones of operation make the cost function discontinuous. In addition to this, constraints such as ramp rate limits, transmission line losses and tie line power distribution effects should be considered for closely estimating actual generator behavior [4–6]. However, these constraints make the task of Economic Dispatch even more complex. Nowadays, due to increasing concern about the environment, the noxious gases emitted from the generators by burning fuels are also being taken into account in the cost function [6]. As a result, the optimization problem in Economic Dispatch consists of multiple goals. The task is to find a load allocation for the generator set such that both the generation costs as well as the emissions are considerably reduced.

A number of classical optimization techniques like Linear Programming (LP) [7], Homogeneous Linear Programming (HLP) [8], Gradient Search algorithm [9], Lambda Iteration method [10], Pattern Search (PS) method [11], Quadratic Programming (QP) method [12] and Sequential Quadratic Programming, method of Lagrangian Multipliers [13] and Non-Linear Programming (NLP) [14] have been applied to solve Economic Dispatch problems. The major problem with these classical optimization techniques is that the final solution returned by them is highly dependent on the position

\* Corresponding author. Tel.: +91 9829227268.

E-mail addresses: [tanujsen.bits@gmail.com](mailto:tanujsen.bits@gmail.com) (T. Sen), [mathurhd@pilani.bits-pilani.ac.in](mailto:mathurhd@pilani.bits-pilani.ac.in) (H.D. Mathur).

of the starting point, which may lead to the problem of trapping in local minima for highly non-linear cost functions [15,16]. The discontinuous nature of the cost function also severely impacts the performance of NLP. Although the Dynamic Programming (DP) Technique can handle discontinuous non-linear cost functions [17], the dimensionality of the optimization problem is a serious concern for it [26].

Due to the limitations of the classical techniques, an increased emphasis is now being given to techniques possessing artificial intelligence. Artificial Neural Networks (ANN) has been successfully applied to solve the Economic Dispatch problem along with the practical constraints such as ramp rate limits, prohibited zones and tie lines [18]. However, a major drawback of this technique is that it is system specific. If the method is to be applied to some different power system, the network needs to be properly tuned by training which takes a long time. In [19], a Riemannian subgradient algorithm with valve point effect is considered. Han et al. [20] have proposed two new solution methods for Dynamic Economic Dispatch. Global optimization techniques such as Genetic Algorithm (GA) [21], Tabu Search (TS) [22], Gravitational Search Algorithm [23], Bio-geography based Optimization [24] and Simulated Annealing (SA) [25] have also been used for this purpose. However, these techniques have some drawbacks regarding their ability to converge toward the final solution which has limited their usage.

The application of Evolutionary Algorithms (EA), which emulate the behavior of swarms, to Economic Dispatch has recently gained much attention. Ant Colony Optimization (ACO) [26], Artificial Bee Colony (ABC) optimization [27] and Particle Swarm Optimization (PSO) [5,28,6] are some such techniques which have been successfully applied to Economic Dispatch problems. Another meta-heuristic technique called the Harmonic Search (HS) algorithm has also been used extensively in this regard [29,30]. The Harmonic Search algorithm is inspired from the technique musicians use to modify tones in order to create a new harmony. The main advantages of these algorithms is that they are search the entire solution space for the best solution and as a result are free from getting trapped in a local minimum and are also able to differentiate between solutions based on their quality. These algorithms are also independent of the nature of the function (they are not concerned with the gradient of the function) allowing their performance to be unaffected by the occurrence of discontinuities in the cost function as compared to the classical optimization techniques. Recently, a number of hybrid methods have been proposed which include the advantages of several independent algorithms into a single framework to provide better results [3,31,32].

In this paper, a hybrid method is proposed for dealing with the problem of Economic Dispatch. The method is an amalgamation of three EA's namely ACO, ABC and HS and is named the Hybrid ACO-ABC-HS algorithm. The algorithm's performance is tested for a ten generator system, accounting for all practical constraints such as valve point loading, environmental emission levels, prohibited zones, transmission losses and tie line considerations. The results provided by the hybrid algorithm are compared with those obtained by using other heuristic techniques namely ABC, ACO, HS and PSO and the classical optimization technique of Gradient Ascent/Descent Optimization.

## Economic Dispatch

Economic Dispatch is concerned with finding out the optimum load allocation to the generators in a thermal power system, which reduces the total fuel cost to the least possible value. In order to find the total fuel cost associated with the power system, we need

to consider the fuel cost function of each generator. For simplicity, the fuel cost function of a generator may be considered as a smooth second order quadratic function of the power generated by the generator, having the form:

$$C(P_{G_i}) = a_i P_{G_i}^2 + b_i P_{G_i} + c_i \quad (1)$$

However, such a cost function cannot be considered for practical cases. Thermal generators generally use steam turbines which suffer from valve point loading effects arising due to the sequential opening of steam valves. When these effects are taken into account, the cost function of the generator ceases to be linear and exhibits ripples of a rectified sinusoidal nature. The modified fuel cost function is modeled as follows:

$$C(P_{G_i}) = a_i P_{G_i}^2 + b_i P_{G_i} + c_i + |e_i \sin(f_i(P_{G_i}^{Min} - P_{G_i}))| \quad (2)$$

There are also a number of constraints which have to be taken into account when solving the problem of Economic Dispatch for multi-generator systems.

1. Environmental emissions: A major problem with thermal generators is the amount of noxious gases they emit into the atmosphere as a result of burning fuels. With growing concern about the environment, it becomes increasingly important to limit the emission of greenhouse gases, particularly NO<sub>2</sub>. The amount of gaseous waste emitted by the generator depends directly on the power generated by it and can be estimated quite accurately by a second order quadratic equation which can be given as follows:

$$E(P_{G_i}) = a_{ei} P_{G_i}^2 + b_{ei} P_{G_i} + c_{ei} \quad (3)$$

2. Transmission losses: The total transmission loss is an important constraint of the Economic Dispatch problem. Not only is it desired that the losses incurred in the system be minimized along with the total fuel cost, but the system must also generate enough power to satisfy the load demand as well as to compensate for the transmission losses. The total losses in the system can be computed using the following relation:

$$P_L = P_G^T B_{ij} P_G + B_{lo} P_G + B_{oo} \quad (4)$$

$$P_G = [P_{G_1} P_{G_2} P_{G_3} P_{G_4} \dots \dots \dots P_{G_n}]^T \quad (5)$$

The power supplied by the individual generator units are combined to form a column vector represented by  $P_G$ . This column vector is used to calculate the total transmission losses.

The power generated by the entire system of generators must be equal to the sum of the total power demand and the losses incurred inside the system and due to transmission.

$$\sum_{i=1}^n P_{G_i} = P_D + P_L \quad (6)$$

If the total power generated by the system (after taking losses into consideration) is not equal to the total power demand, the system will be unbalanced and this can seriously affect the grid voltage and frequency. Therefore, the cost function for the thermal power system after taking into consideration all the above mentioned constraints is given as:

$$F(P_G) = \sum_{i=1}^n C(P_{G_i}) + \mu_1 \left( \sum_{i=1}^n E(P_{G_i}) \right) + \mu_2 \left( \sum_{i=1}^n P_{G_i} - (P_D + P_L) \right)^2 \quad (7)$$

where  $\mu_1$  and  $\mu_2$  are the weight factors which determine the contribution of the emissions constraint and the load balance constraints to the cost function respectively. These two additional terms in the cost function serve as penalty factors which penalize a candidate solution if it does not satisfy the aforementioned constraints.

3. Region of operation: Every thermal generator has a specified range within which its operation is stable. Therefore, it is desired that the generators be run within this range in order to maintain system stability.

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max} \quad (8)$$

4. Prohibited zones: There may be zones of operation within the specified range wherein the system may lose stability. These are termed as prohibited zones and in practical operation; these regions are usually avoided during generation.

$$P_{G_i} \in \begin{cases} P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_{i,1}}^l \\ P_{G_{i,1}}^u \leq P_{G_i} \leq P_{G_{i,2}}^l \\ \dots \\ P_{G_{i,n}}^u \leq P_{G_i} \leq P_{G_i}^{\max} \end{cases} \quad (9)$$

where  $P_{G_{i,n}}^u$  and  $P_{G_{i,n}}^l$  denote the upper and lower limits of the  $n$ th prohibited zone. The prohibited zones make the cost function discontinuous in nature.

5. Generator ramping rates: For any sudden change in the load, generators increase or decrease their supply in order to reduce the power mismatch to zero. However, the generators can change their power supply only at a certain rate determined by the up-ramping and down-ramping rate. If a generator is operating at a specific point, then its point of operation can be changed only up to a certain limit determined by the ramping rate. Therefore, for closely approximating the practical case, the ramp rate limits also need to be taken into consideration when dealing with problem of Economic Dispatch.

For increase in generation:

$$P_{G_i} + \Delta P_{G_i} \leq \min(P_{G_i} + UR_i, P_{G_i}^{\max}) \quad (10)$$

For decrease in generation:

$$P_{G_i} - \Delta P_{G_i} \geq \max(P_{G_i} - DR_i, P_{G_i}^{\min}) \quad (11)$$

6. Transmission line flow limits: When considering the Multi-Area Economic Dispatch (MAED) problem, we have to control the amount of power flowing from one area to another. This is because the transmission lines have a specific power carrying capacity. Exceeding these limits may prove detrimental to the system. The above mentioned requirement can be framed as an inequality constraint.

$$P_{Line_i} \leq P_{Line_{max}}, \quad i = 1, 2, \dots, n_L \quad (12)$$

where  $n_L$  is the number of transmission lines.

## Proposed algorithm

A new hybrid algorithm is presented in this paper for the purpose of Economic Dispatch. This hybrid algorithm capable of finding the optimal generator allocation for a thermal power system, taking into account all the practical constraints as discussed above (see Fig. 1). The proposed hybrid algorithm was formulated by amalgamating the features of three heuristic algorithms namely the Ant Colony Optimization (ACO), the Artificial Bee Colony (ABC) optimization and the Harmony Search (HS) optimization. Hence, Hybrid ACO-ABC-HS has been chosen as the name of the

proposed algorithm. Due to limitations of the length of the paper, only the detailed analysis of the proposed algorithm has been provided.

The first step involved in implementing the Hybrid ACO-ABC-HS algorithm is to initialize the solution set such that the candidate solutions are spread over the entire search space. The solutions are initialized in a way similar to that adopted in the ACO algorithm. The generators are given a random load allocation which lies within their operating limits and avoids the prohibited zones. The generators are selected at random and a part of the total demand is assigned to them. If the total demand is satisfied by loading a few generators (taking losses into consideration), the rest are left unloaded, thus saving fuel as well as reducing the total cost incurred. The load allocated to be handled by a generator is decided by using Eq. (13).

$$P_{G_i} = P_{G_i}^{\min} + \emptyset(P_{G_i}^{\max} - P_{G_i}^{\min}) \quad (13)$$

$\emptyset$  is random number between 0 and 1 [33]. Once the candidate solutions are produced, the execution of the main algorithm is initiated with makes use of the Employed and Onlooker Bees concept of the ABC algorithm [34]. First, the employed bees visit all the candidate solutions produced earlier and evaluate their fitness. Next they search the neighborhood of the candidate solution to look for better solutions. The employed bees update their position using the following equation:

$$x_{i,j}^{new} = x_{i,j}^{old} + \varphi(x_{k,j} - x_{i,j}^{old}) \quad (14)$$

where  $i$  denotes the  $i$ th solution among all the candidates and  $x_k$  is a randomly selected solution ( $k \neq i$ ).  $j$  denotes one random dimension selected out of the possible  $D$  dimensions.  $\varphi$  is a random number ranging between  $-1$  and  $1$ . If the new solution produced satisfies all the constraints mentioned in the previous section and is better than the solution with the worst fitness value present among the candidate solutions, the new solution replaces that solution. This technique of solution update is the same as that followed in the Harmonic Search (HS) algorithm [35].

Next, the probability of selection of the solutions by the Onlooker Bees has to be determined. For this purpose, the Pheromone deposition Technique of the ACO algorithm is selected. The amount of pheromone assigned to one solution is directly dependent on its fitness and is given by the following equation:

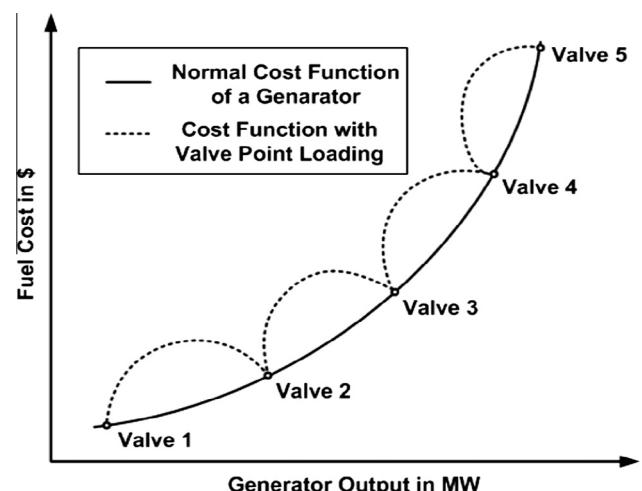


Fig. 1. Fuel cost curve representation with and without valve point loading effects.

$$p_i^k = \frac{(\tau_i^\alpha)(\eta_i^\beta)}{\sum_{i=1}^n (\tau_i^\alpha)(\eta_i^\beta)} \quad (15)$$

Here  $\tau_i$  denotes the amount of pheromone assigned to the  $i$ th solution and  $\eta_i$  is a value related to the fitness of the  $i$ th solution (mostly taken as the inverse of the solution's fitness).  $\alpha$  and  $\beta$  are constants which decide the contribution of the above two factors to the probability of selection. The pheromone values are updated every iteration in a manner similar to that followed in the ACO algorithm. A solution having a higher probability of selection is more likely to be selected by the Onlooker Bees, which in turn improve them by using the same update Eq. (14) as used by the Employed Bees. The improved solutions again replace the worst solution among all the candidate solutions, provided they satisfy all the necessary constraints.

If a solution cannot be improved any further, then it is discarded and a new solution is selected from the search space, using the same technique (13) as described earlier to initialize the candidate solutions. This generation of new solutions to replace obsolete ones mimics the functioning of Scout Bees in the ABC algorithm. This entire process is repeated for the specified number of iterations after which the algorithm returns the best solution found by it over the entire duration of its execution. The entire process is presented in the form of a flowchart in Fig. 3.

### System description

A ten generator system is considered for testing the performance of the proposed algorithm. The fuel cost function coefficients (including valve point loading effects), emission level function coefficients as well as the operable region of the generators are given in Table 1.

For the ten generator system, it is considered that five of them have prohibited zones within their region of operation. The prohibited zones of these five generators are mentioned in Table 2.

The transmission line loss coefficient matrices for the ten generator system are given as follows:

$$B_{ij} = 0.01 * \begin{bmatrix} 0.0008 & 0.0009 & 0.0015 & 0.0013 & 0.0012 & 0 & 0 & 0.0007 & 0.0006 & 0.0011 \\ 0.0004 & 0.0021 & 0.0018 & 0.0005 & 0.0007 & 0.0010 & 0.0013 & 0.0008 & 0.0001 & 0.0016 \\ 0.0004 & 0.0017 & 0.0015 & 0.0005 & 0.0006 & 0.0011 & 0.0007 & 0.0003 & 0.0004 & 0.0007 \\ 0.0005 & 0.0021 & 0.0008 & 0.0009 & 0.0009 & 0.0017 & 0.0015 & 0.0009 & 0 & 0.0023 \\ 0.0007 & 0 & 0.0008 & 0.0005 & 0.0003 & 0 & 0.0014 & 0.0009 & 0.0019 & 0 \\ 0.0014 & 0.0006 & 0 & 0.0006 & 0.0009 & 0.0011 & 0.0001 & 0.0005 & 0 & 0.0003 \\ 0 & 0.0001 & 0.0010 & 0.0005 & 0.0006 & 0.0015 & 0.0012 & 0.0018 & 0.0004 & 0.0005 \\ 0.0017 & 0.0013 & 0.0008 & 0.0016 & 0.0019 & 0.0008 & 0.0015 & 0.0018 & 0.0014 & 0.0006 \\ 0.0007 & 0.0001 & 0.0019 & 0.0011 & 0.0008 & 0.0012 & 0.0006 & 0.0018 & 0.0022 & 0.0003 \\ 0.0007 & 0.0010 & 0.0016 & 0.0014 & 0.0005 & 0.0013 & 0.0007 & 0.0008 & 0.0003 & 0.0012 \end{bmatrix}$$

$$B_{lo} = 0.001 * [0.287 \ 0.012 \ 0.0896 \ 0.1471 \ 0.0087 \ 0.3121 \ 0.233 \ 0.1123 \ 0.0912 \ 0.1121]$$

$$B_{oo} = 0.038$$

For evaluating the performance of the proposed algorithm for a Multi Area Economic Dispatch problem, the same generator set was divided into three separate local generation areas connected to each other via tie lines. A diagrammatic elucidation of the multi area system is given in Fig. 2. Generators #1, #2 and #3 make up

Local Area 1. Local Area 2 comprises of Generators #4 to #7 while Generators #8, #9 and #10 form Local Area 3. Each of these local areas is connected to the others by tie lines.

### Simulation results

A number of case studies were conducted to validate the superior performance of the proposed ACO-ABC-HS algorithm in solving optimization problems related to Economic Dispatch with non-smooth cost functions. The results provided by the proposed algorithm were compared with other meta-heuristic algorithms which have been used widely for solving Economic Dispatch problems, namely ABC, ACO, HS and PSO and the classical Gradient Search method. Simulations have been conducted for cases involving Isolated Power Systems and Interconnected Power Systems. All simulations were conducted in MATLAB on a 2.1 GHz Intel Core i7 processor with 8 GB RAM. The results provided are an average of 50 independent runs to account for the variations in the results provided by the stochastic algorithms.

#### Single Area Economic Dispatch (Isolated Power System)

The simulation results obtained for an Isolated Power System consisting of ten generators are presented first. The simulations have been performed for three different cases of load demand. All the algorithms have been run for 200 iterations and the results obtained over 50 test runs have been averaged to provide the final result. In order to provide a fair comparison of the algorithms' performances, the same initial solution set has been used by all the algorithms as the starting reference.

- i. Case-I: The optimal dispatch was first determined for the case when the system had to handle a load of 300 MW. The averaged results obtained by applying all the aforementioned algorithms are presented in the table below (see Table 3).

From the results, it is evident that the hybrid algorithm provides the optimal dispatch schedule, ensuring minimum fuel cost. Although the dispatch obtained by using the PSO algorithm provides the lowest emissions and that provided by the Gradient Search method give the least line losses, the

**Table 1**

Fuel Cost Function coefficients, Emissions coefficients and Operating ranges of the generators.

Generator unit	$a_i$	$b_i$	$c_i$	$e_i$	$f_i$	$a_{ei}$	$b_{ei}$	$c_{ei}$	$P_{MAX}$ (MW)	$P_{MIN}$ (MW)
1	0.591	21.34	45.781	54.63	0.031	0.0075	0.121	13.561	100	12.5
2	0.561	11.45	89.234	31.24	0.04	0.00432	0.233	19.256	100	13
3	0.34	21.67	56.783	63.37	0.087	0.00116	0.137	16.096	100	10
4	0.2533	9.013	109.6	89.45	0.012	0.0064	0.343	19.437	120	15
5	0.0864	7.09	77.345	56.12	0.008	0.00453	0.149	17.674	125	12
6	0.081	17	125	45.29	0.076	0.0018	0.126	25.631	125	14
7	0.08	12.34	90.4	73.63	0.036	0.0083	0.181	19.442	130	20
8	0.074	27.89	157.43	61.90	0.009	0.0015	0.209	27.431	135	25
9	0.0471	16.5	88.56	49.14	0.057	0.0023	0.151	18.462	150	17.5
10	0.025	13.5	134	65.87	0.078	0.0034	0.115	13.412	150	20

**Table 2**

Prohibited zones of operation.

Generator unit	Prohibited zones (MW)	
	Zone 1	Zone 2
1	[50 58]	None
4	[60 70]	[95 107]
6	[36 43]	[100 109]
7	[115 120]	None
10	[52 58]	[121 130]

cost incurred for these dispatches, especially the total cost which takes into account the fuel cost, the emissions, the transmission losses and surplus power generation, is extremely high. Among the others, the hybrid algorithm provides the best overall result. The total power supply is the lowest for the hybrid algorithm. It can also be noted that the algorithms have been framed in such a way that they can decide which generators can be left unloaded, thus helping to reduce the total cost.

- ii. Case-II: The same system was required to satisfy a load demand of 500 MW and the optimal dispatch results are presented in Table 4.

It can be noticed from the results that the hybrid algorithm again provides the most Economic Dispatch. The losses for the dispatch obtained by ABC and Gradient Search are lower, but the total cost incurred for these cases are higher.

- iii. Case-III: Another simulation was also carried out for the case when the system had to handle a load of 700 MW. The results are provided in Table 5. The results show that the hybrid algorithm again provides the most Economic Dispatch. Thus, it is proven that the hybrid algorithm is indeed better than the other algorithms discussed for finding the optimal dispatch for an Isolated Power System.

#### Multi-Area Economic Dispatch (Interconnected Power Systems)

To test the robustness of the proposed algorithm, it was applied for solving the Economic Dispatch problem for multiple supply areas connected to each other via tie lines. In MAED problems, each area needs to satisfy its own individual demand in addition to the total power demand. The ten generator system presented earlier is now divided into 3 separate areas, Area #1 having Generators #1, #2 and #3, Area #2 having Generators #4, #5, #6 and #7 and Area #3 possessing the rest. In these cases, tie line considerations and line capacity limits are also to be taken into account. For the cases presented, the line capacity has been taken to be 100 MW for all tie lines and all tie lines have equal cost coefficients. Comparisons have also been made with the same algorithms which had been used for the Single Area case. The results present the average of 50 independent trials with each algorithm being run for 200 iterations.

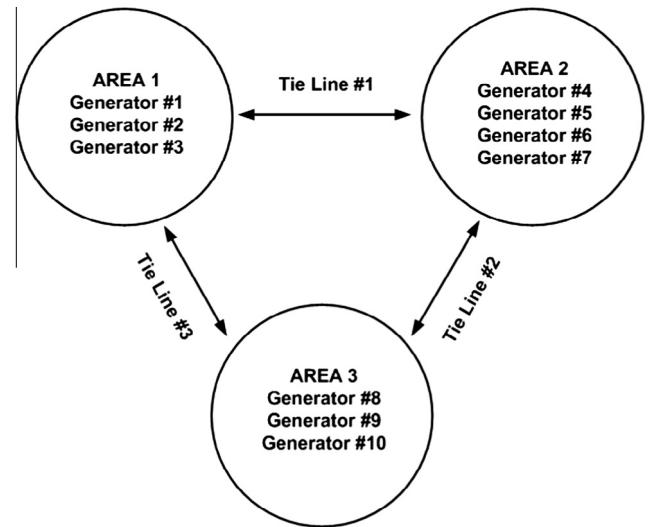
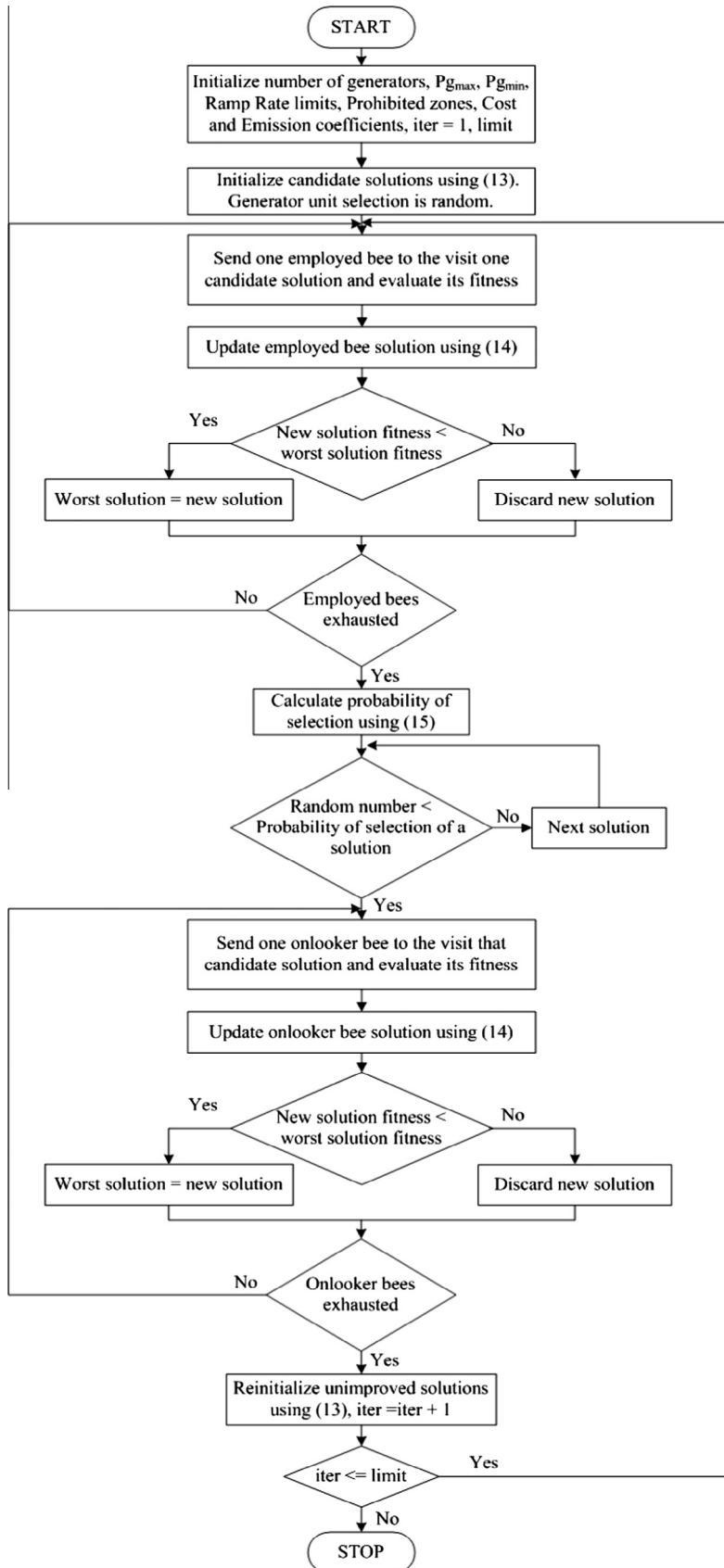


Fig. 2. Diagrammatic depiction of the interconnected power system used for conducting the simulations for Multi-Area Economic Dispatch.

- i. Case-I: A demand of 450 MW was placed on the system with Area #1 requiring 100 MW, Area #2 requiring 200 MW and Area #3 having a demand of 150 MW. The emissions, transmission losses, fuel costs and total costs are presented along with the results in Table 6. The results suggest that not only does the hybrid algorithm provide the most optimal dispatch; it is also able to satisfy the demands of each area satisfactorily. The hybrid algorithm is also able to recognize the generator which can be left unloaded in order to decrease fuel costs.
- ii. Case-II: Next, the system was subjected to a total demand of 600 MW with the individual demands of Areas #1, #2 and #3 being 150 MW, 225 MW and 225 MW. The results are provided in Table 7, which again prove that the hybrid algorithm generates the most Economic Dispatch for this case too. Although the PSO and Gradient Descent algorithms are able to produce dispatches wherein the emissions are lower, their cost efficiency is not comparable to the results generated by the other algorithms. Another point to be noted is that the classical Gradient Search method fails miserably to satisfy the demands of individual areas, thereby highlighting the limitations of classical algorithms for MAED problems.
- iii. Case-III: For the final test case, the system was required to satisfy a demand of 800 MW with 200 MW, 325 MW and 275 MW being the individual area demands. The results in Table 8 again suggest the superiority of the Hybrid ABC-ACO-HS algorithm over the others in finding the optimal dispatch.



**Fig. 3.** Flowchart describing the sequence of execution of the proposed algorithm.

**Table 3**

Optimal dispatch results for a load demand of 300 MW.

Generator unit	ABC	ACO	HS	ACO–ABC–HS	PSO	Gradient search
1	12.5000	0	12.5000	12.5000	0	0
2	13.0000	13.0000	13.0000	13.0000	0	0
3	10.0000	10.0000	10.0000	10.0000	20.4046	18.5600
4	21.6472	26.0315	21.5043	20.5575	15.0000	15.0000
5	71.8755	65.7131	71.7938	74.3546	65.2755	125.0000
6	14.0000	16.9004	14.0000	14.0000	44.0719	34.9027
7	35.7725	33.8452	35.6603	31.9622	20.0000	68.7361
8	0	0	0	0	29.7851	0
9	21.9071	37.4972	22.2852	22.5972	17.5000	0
10	100.5558	101.0050	100.5144	101.9651	96.1525	44.9138
Total supply	301.2581	303.9924	301.258	300.9366	308.1896	307.1126
Emissions (lbs)	<b>312.435</b>	<b>311.741</b>	<b>312.257</b>	<b>312.254</b>	<b>298.677</b>	<b>359.606</b>
Total losses (MW)	<b>0.781527</b>	<b>0.816283</b>	<b>0.781272</b>	<b>0.773257</b>	<b>0.849413</b>	<b>0.730886</b>
Fuel cost (\$/h)	<b>5988.007</b>	<b>5998.743</b>	<b>5988.575</b>	<b>5986.806</b>	<b>6663.317</b>	<b>6854.187</b>
Total cost (\$/h)	<b>6322.985</b>	<b>7318.524</b>	<b>6323.396</b>	<b>6301.693</b>	<b>12343.774</b>	<b>11274.296</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

**Table 4**

Optimal dispatch results for a load demand of 500 MW.

Generator unit	ABC	ACO	HS	ACO–ABC–HS	PSO	Gradient search
1	12.5000	12.5000	12.5000	12.5000	12.5000	13.4868
2	14.4018	13.0000	13.0000	13.0000	14.4134	13.5586
3	10.1410	10.0000	10.0000	10.0000	10.0069	12.2281
4	18.7771	25.4099	26.1563	26.0157	26.3741	18.3283
5	90.7798	94.7287	87.8938	87.5698	87.5469	125.0000
6	56.4372	17.3114	55.2734	55.1233	50.5875	20.6141
7	44.6637	81.1636	58.9697	59.5171	86.3165	93.8312
8	0	25.0000	25.0000	25.0000	28.0985	25.0000
9	104.4135	73.7456	72.6199	72.6158	75.9916	34.5543
10	150.0000	150.0000	140.8444	140.8693	110.7603	150.0000
Total supply	502.1141	502.8592	502.2575	502.211	502.5957	506.6014
Emissions (lbs)	<b>430.479</b>	<b>461.588</b>	<b>424.674</b>	<b>424.887</b>	<b>434.797</b>	<b>498.341</b>
Total losses (MW)	<b>2.096248</b>	<b>2.187990</b>	<b>2.170972</b>	<b>2.170853</b>	<b>2.213229</b>	<b>2.115520</b>
Fuel cost (\$/h)	<b>10440.658</b>	<b>10480.893</b>	<b>10424.557</b>	<b>10423.280</b>	<b>10631.363</b>	<b>10836.733</b>
Total cost (\$/h)	<b>10871.208</b>	<b>10987.020</b>	<b>10849.953</b>	<b>10848.324</b>	<b>11080.484</b>	<b>13340.514</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

**Table 5**

Optimal dispatch results for a load demand of 700 MW.

Generator unit	ABC	ACO	HS	ACO–ABC–HS	PSO	Gradient search
1	12.5000	12.5000	12.5000	12.5000	12.5000	14.1900
2	14.7023	13.0000	15.6206	15.2339	13.0000	28.2523
3	10.0000	10.0000	10.0000	10.0000	10.1118	17.2527
4	41.4472	44.8773	37.6257	38.1130	35.2511	28.6908
5	125.0000	113.0877	123.6881	121.8794	112.6061	125.0000
6	81.7286	92.8989	95.3495	96.5682	87.5651	60.2893
7	107.5818	104.8606	106.5685	107.2279	113.4063	100.1269
8	25.0000	30.5417	25.0000	25.0000	27.6828	33.2657
9	136.9440	133.6212	127.8728	127.6973	142.3099	150.0000
10	150.0000	150.0000	150.0000	150.0000	150.0000	150.0000
Total supply	704.9039	705.3854	704.2252	704.2196	704.4331	707.0677
Emissions (lbs)	<b>615.130</b>	<b>601.656</b>	<b>607.724</b>	<b>607.242</b>	<b>613.474</b>	<b>603.120</b>
Total losses (MW)	<b>4.268920</b>	<b>4.322704</b>	<b>4.216306</b>	<b>4.219579</b>	<b>4.292509</b>	<b>4.288977</b>
Fuel cost (\$/h)	<b>15760.136</b>	<b>15808.595</b>	<b>15709.136</b>	<b>15705.205</b>	<b>15802.084</b>	<b>16081.452</b>
Total cost (\$/h)	<b>16415.264</b>	<b>16522.918</b>	<b>16316.891</b>	<b>16315.367</b>	<b>16417.555</b>	<b>17418.269</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

**Table 6**

Optimal dispatch results for a load demand of 450 MW (Area 1: 100 MW, Area 2: 200 MW, Area 3: 150 MW).

Generator unit	ACO	ABC	HS	ACO-ABC-HS	PSO	Gradient Search
Area #1	1	34.4504	23.3239	21.2422	21.3917	22.5833
	2	34.6466	30.4603	32.5723	32.5067	25.8168
	3	31.2359	44.3928	46.2027	46.1030	50.7074
Area #2	4	16.1979	34.4172	27.0148	28.8352	29.6615
	5	62.8116	84.7612	75.0395	73.4269	74.7903
	6	14.0000	26.8362	55.3122	55.3393	50.3926
	7	109.1896	53.9669	42.8062	42.4169	46.7068
Area #3	8	26.0708	25.0000	25.0000	0	27.3384
	9	17.5000	26.8183	68.2016	53.0394	90.3496
	10	106.8847	105.1116	60.2224	100.5548	41.1300
Total supply		452.9875	455.0884	453.6139	453.6139	459.4767
Area #1 supply		100.3329	98.177	100.0172	100.0014	99.1075
Area #2 supply		202.1991	199.9815	200.1727	200.0183	201.5512
Area #3 supply		150.4555	156.9299	153.424	153.5942	158.818
Emissions (lbs)		<b>438.393</b>	<b>381.478</b>	<b>349.810</b>	<b>363.436</b>	<b>374.611</b>
Total losses (MW)		<b>1.855524</b>	<b>1.869687</b>	<b>1.867201</b>	<b>1.810670</b>	<b>2.002400</b>
Fuel cost (\$/h)		<b>11031.826</b>	<b>10825.561</b>	<b>10834.308</b>	<b>10508.128</b>	<b>11410.336</b>
Total cost (\$/h)		<b>13345.711</b>	<b>13752.803</b>	<b>12882.113</b>	<b>12650.592</b>	<b>19076.756</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

**Table 7**

Optimal dispatch results for a load demand of 600 MW (Area 1: 150 MW, Area 2: 225 MW, Area 3: 225 MW).

Generator unit	ACO	ABC	HS	ACO-ABC-HS	PSO	Gradient search
Area #1	1	38.5243	36.0933	33.3840	33.7322	12.5000
	2	35.6658	36.9493	46.0244	45.7801	68.5945
	3	75.2601	76.9564	70.6128	70.4892	68.9711
Area #2	4	23.1054	39.3525	30.7427	30.4530	24.6885
	5	109.5841	88.1895	85.3291	86.5209	86.8320
	6	45.9853	56.3149	56.1778	55.3521	73.1267
	7	50.5717	48.0325	52.6971	52.7062	47.9436
Area #3	8	25.0000	25.0000	25.0000	25.0000	70.3129
	9	67.9608	67.3694	66.6959	72.6033	57.0926
	10	132.0721	133.3500	138.0551	131.1438	100.2394
Total supply		603.7296	607.6078	604.7189	603.7808	610.3013
Area #1 supply		149.4502	149.999	150.0212	150.0015	150.0656
Area #2 supply		229.2465	231.8894	224.9467	225.0322	232.5908
Area #3 supply		225.0329	225.7194	229.751	228.7471	227.6449
Emissions (lbs)		<b>460.473</b>	<b>451.457</b>	<b>452.912</b>	<b>449.235</b>	<b>434.374</b>
Total losses (MW)		<b>3.173551</b>	<b>3.310005</b>	<b>3.264674</b>	<b>3.251630</b>	<b>3.466528</b>
Fuel cost (\$/h)		<b>15928.056</b>	<b>15998.359</b>	<b>15784.145</b>	<b>15783.663</b>	<b>17288.219</b>
Total cost (\$/h)		<b>18257.291</b>	<b>20070.773</b>	<b>18249.586</b>	<b>18054.230</b>	<b>24050.516</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

### Convergence rate and execution time comparison

A separate test was conducted to compare the rate of convergence of the algorithms and to measure the time taken by them to converge toward their optimal solutions. The test conducted was for a four area interconnected system with Generators #1 and #8 making up Area 1, Generators #2, #3 and #7 in Area 2, Generators #5, #6 and #10 in Area 3 and the rest in Area 4. The total load demand was set as 500 MW with individual Area demands being 100 MW, 125 MW, 175 MW and 100 MW. The test was run for 100 iterations and the convergence characteristics are presented in Fig. 4. The Cumulative (Total) Cost signifies the total cost incurred including emissions, transmission losses and tie line constraints averaged over 50 separate trials for the same case.

The results show that the proposed hybrid algorithm is the fastest in converging to its final solution and also provides the most

Economic Dispatch for the system. Although the result given by the HS and ABC algorithms are comparable in quality, both algorithms require a greater number of iterations to converge to their final solution. The HS algorithm is the fastest among the other algorithms to converge to the final solution; however, it takes almost 3 times more number of iterations as compared to the proposed algorithm. The HS algorithm converges to the final solution in 61 iterations whereas the hybrid algorithm takes 22 iterations to reach the final solution. The results thus prove the faster convergence characteristics of the proposed algorithm.

Fig. 5 presents a graph of the average time taken by each algorithm to converge to their final solutions. The graph shows that the Hybrid ACO-ABC-HS algorithm takes the most time to execute. This is justified because it is the amalgamation of three independent optimization algorithms which themselves take a considerable amount of time to execute. However, considering the fast

**Table 8**

Optimal dispatch results for a load demand of 800 MW (Area 1: 200 MW, Area 2: 325 MW, Area 3: 275 MW).

Generator unit		ACO	ABC	HS	ACO-ABC-HS	PSO	Gradient search
Area #1	1	48.7790	42.8023	49.0546	49.3621	59.2731	43.9155
	2	80.3103	64.8128	63.8839	63.1373	62.4231	68.8115
	3	70.6556	92.5027	87.0916	87.5018	78.1993	87.4094
Area #2	4	39.2111	17.2841	41.3637	40.0365	78.1809	56.3829
	5	90.6642	114.8824	110.1900	110.4038	66.3653	125.0000
	6	71.7222	94.6338	96.7200	96.6692	85.2245	83.4520
	7	123.4135	100.2713	76.8547	77.9109	95.2433	77.7091
Area #3	8	29.1850	25.6564	25.0000	25.0000	76.5887	45.8771
	9	109.2416	108.8741	106.5854	115.1285	92.4718	72.9656
	10	143.6759	147.0917	150.0000	140.8921	114.4223	145.6610
Total supply		806.8584	808.8116	806.7439	806.0422	808.3923	807.1841
Area #1 supply		199.7449	200.1178	200.0301	200.0012	199.8955	200.1364
Area #2 supply		325.011	327.0716	325.1284	325.0204	325.014	342.544
Area #3 supply		282.1025	281.6222	281.5854	281.0206	283.4828	264.5037
Emissions (lbs)		<b>655.310</b>	<b>620.121</b>	<b>598.320</b>	<b>594.320</b>	<b>601.335</b>	<b>608.733</b>
Total losses (MW)		<b>5.718535</b>	<b>5.689578</b>	<b>5.725589</b>	<b>5.662267</b>	<b>6.109098</b>	<b>5.783842</b>
Fuel cost (\$/h)		<b>23692.909</b>	<b>23454.909</b>	<b>23340.079</b>	<b>23311.178</b>	<b>24795.813</b>	<b>24346.720</b>
Total cost (\$/h)		<b>27084.448</b>	<b>27530.216</b>	<b>26435.373</b>	<b>26293.429</b>	<b>28287.927</b>	<b>27379.240</b>

Bold and shaded values in signify the results provided by the proposed Hybrid algorithm.

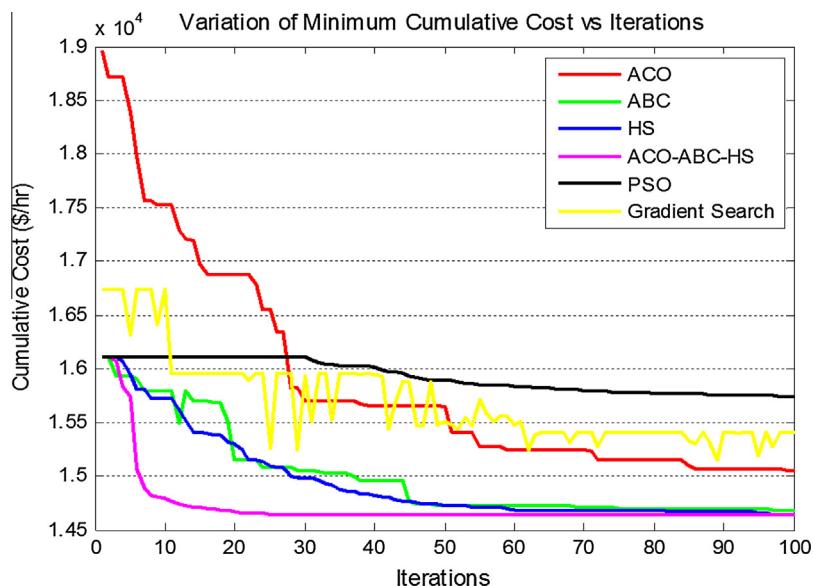


Fig. 4. Rate of convergence of the algorithms when finding the Economic Dispatch for a multi-area interconnected power system.

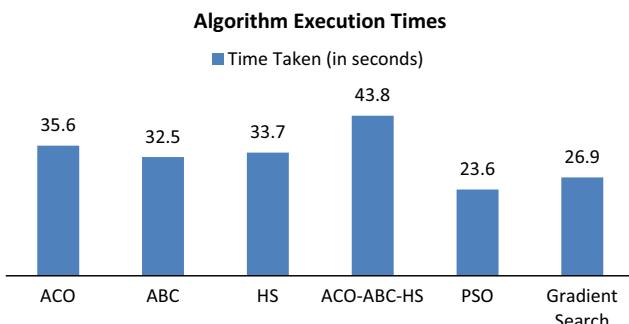


Fig. 5. Graph depicting the average time taken by algorithms (in seconds) to complete execution of 100 iterations.

convergence of the proposed algorithm and the better quality of solution produced by it, the slightly higher execution time of the algorithm is more than compensated. The PSO and Gradient Search algorithms take the least amount of time to complete execution. However, the quality of their final solutions is not comparable to that of the proposed method.

The main motive of the proposed optimization algorithm is to reach the best final solution in the minimum possible time. Although Fig. 5 shows that the proposed hybrid algorithm takes the longest amount of time to execute, it should be noted that the data provided is for a standard of 100 iterations. From Fig. 4, it is also clear that the proposed algorithm requires the least number of iterations to converge to the final solution. Combining the

two sets of information obtained from Figs. 4 and 5, we find that the hybrid algorithm takes the least amount of time to converge to the final solution. Through straightforward calculations, it can be found out that the hybrid algorithm takes around 9.636 s (time taken to execute 22 iterations) to converge to the final solution. All other algorithms require a much longer time to reach their respective solutions.

Another striking factor about the hybrid algorithm is the quality of solutions provided by it. The hybrid algorithm is able to provide a dispatch schedule which is the most economic, while reducing emissions and line losses to a bare minimum. The other algorithms are able to provide similar solutions for a Single Area Economic Dispatch problem. However, in Multi-Area Economic Dispatch problem, the conventional algorithms, especially the PSO and the Gradient Descent algorithms, face problems with satisfying the demand of each individual area in addition to providing the most Economic Dispatch. The hybrid algorithm is effectively able to mitigate this problem as can be seen through the simulation results.

## Conclusion

This paper presents a new hybrid algorithm to solve the problem of Economic Dispatch. The algorithm is an amalgamation of three well known meta-heuristic algorithms namely the ACO, ABC and HS and hence it is named the ACO-ABC-HS algorithm. Several tests were conducted for different power system configurations in order to test the effectiveness of the proposed algorithm. Constraints such as valve point loading effects, environmental emissions, transmission line losses, ramp rate limits, prohibited zones, tie line considerations and line capacities have been considered in order to emulate practical conditions for the system. The results obtained by the application of the proposed algorithm indicate that it is indeed very successful in tracking and finding the optimal dispatch. The results provided by the hybrid algorithm have also been compared with those provided by other meta-heuristic and classical algorithms in order to validate its better performance. A comparative analysis reveals the superior quality of the solutions provided by the hybrid algorithm in addition to other advantageous features like fast convergence rate, ability to handle discontinuous cost functions and the like. The algorithm works perfectly for isolated as well as Interconnected Power Systems, thereby ensuring compatibility to handle any power system configuration.

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