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A generalized approach for determination of optimal location and performance analysis of FACTs devices

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ABSTRACT

This paper presents a generalized approach for determination of optimal locations for placement of Flexible AC Transmission Systems (FACTs) devices in the power system with an objective of reducing real power loss and to reduce overloading of the lines. An objective function involving above objectives is formulated and a detailed mathematical model for each objective is presented in terms of system parameters. Three FACT devices, namely, Unified Power Flow Controller (UPFC), Interline Power Flow Controller (IPFC), and Optimal Unified Power Flow Controller (OUPFC) which are capable of controlling both active and reactive power are considered in simulation and analysis of the networks. The parameters to be optimized have been identified and incorporated in the objective function for each device. Sensitivity analysis is used to locate optimal buses to place the FACTs devices in the network. Effectiveness of the approach is

Introduction

There is an enormous increase in the power transactions due to power system restructuring. Right-of-way, environmental, and high cost are major hurdles for power transmission network expansion, which necessitates the need to explore the unused potential of the transmission system capacity. FACT devices can reduce system losses, increase power transfer capability and stability.

There are various approaches proposed in the literature for optimizing location and parameter settings of the FACT devices. Unified power flow controller (UPFC), Interline Power Flow Controller (IPFC), and OUPFC can be used to change power flow in the lines by changing their parameters to achieve various objectives. FACT devices can control steady state power flow as well as system parameters in dynamic state [1–4]. Network, power flow can be controlled by placing FACT devices in appropriate locations [5] without changing the generation schedule and topology of power system. There is an increased interest in FACTs due to the advancements in modern power electronic devices [6] combined with deregulation of power industry. Power flow control is a cost-effective means of dispatching specified power transactions. Congestion in a transmission system cannot be permitted for long duration which otherwise results in cascade outages.

UPFC can be used for direct bus and line voltage control, series compensation, phase shifter and their combinations. UPFC combines properties of series and shunt controllers. It is a two-converter series-shunt FACT controller which has better power flow and voltage control capability than one-converter FACTs controller.

The UPFC [7] is one of the most promising FACTs devices for load flow control since it can either simultaneously or selectively control active and reactive power flow along the lines as well as the nodal voltages [8–11].

IPFC combines two or more series FACT controllers. The UPFC can control power flow of only one transmission line. Whereas IPFC can control power flows of a group of lines and sub-networks. The IPFC scheme provides, independently controllable reactive series compensation for each individual line. It has a capability to directly transfer real power between compensated lines. This capability makes it possible to equalize both real and reactive power flow between the lines and to transfer power demand from over loaded to under loaded lines [12].

Another popular device, The OUPFC, combines a conventional Phase Shifting Transformer (PST) and a UPFC as a better cost-effective device compared to a standalone UPFC [13]. A steady-state model of the OUPFC and its operational characteristics were introduced in [13]. PST controls and transfers the power through certain paths. PST models and operational characteristics are well established in [14–17].

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0142-0615/

The optimal choice and allocation of FACT devices are very important since installation of FACT devices in any power system has an investment constraint. It can offer additional opportunities for operational improvement through integration of economic and technical objectives. The reliable and secure operation of power systems is an important task for operators to supply the electrical energy demand under normal and contingency conditions.

A parallel Tabu search based placement of UPFC for enhancement of Available Transfer Capability (ATC) has been proposed in [18]. Lima et al. [19] proposed number, network location, and settings of phase shifters to maximize system loadability using mixed integer linear programming (MILP). UPFC is placed to improve the transfer capability in the transmission systems [20]. Eigen vector analysis is used for placement of SVC and TCSC in [21]. The installation of FACT controllers to improve steady state security of power system has been reported in [22].

The impact of controllers on ATC is studied in [23], where FACT controllers are placed using GA for improving voltage profile and total transfer capability of the system. Kumar et al. [24] proposed MILP approach for combined optimal location of FACT controllers for loadability enhancement in hybrid electricity markets. In competitive environment, the system loadability has been calculated in [25].

IPFC [26,12] belongs to the conceptual framework of the convertible static compensator (CSC). IPFC significantly controls power flow of multi lines or a sub-network rather than control the power flow of a single line by a UPFC [4,7,40] or static synchronous series compensator (SSSC) [27], or voltage control by a static synchronous (shunt) compensator (STATCOM) [28].

The multi objective evolutionary algorithm (EA) has been [illegible]

[illegible] swarm optimization (PSO) technique was used for optimal location of the thyristor-controlled series capacitor (TCSC), static VAR compensator (SVC), and UPFC with a minimum cost of installation and to improve the system loadability [31]. The optimal location of the TCSC, SVC, and static synchronous compensator (STATCOM) is determined based on fuzzy decision making and the genetic algorithm (GA) [32].

A Linear Programming (LP)-based Optimal Power Flow (OPF) algorithm is used for corrective FACT control to relieve overloads and voltage violations and to minimize average loadability on heavily loaded energy transmission lines [33]. Optimal location and setting of TCSC under a single line contingency, using MINLP, are presented in [34]. System static security has been enhanced via optimal placement of TCSC to alleviate overloads during single contingencies [35]. The optimal location of STATCOM and SVC, based on contingency voltage stability, has been studied using Continuation Power Flow (CPF) [36]. The steady-state model of OUPFC and its operational characteristics have been introduced in [13]. The performance of UPFC was compared with SVC and phase shifter in [37].

The optimization framework is mathematically modeled as Non-Linear Programming (NLP) and solved using a CONOPT solver in the General Algebraic Modeling System (GAMS) [38,39].

The proposed work is an extension of [45] in which the same generalized approach has been implemented to determine optimal Location of TCSC and TCPAR.

There are several methods to find optimal locations of specified type of FACT device. However, there is no generalized approach for placement of any type of FACT device. This paper presents a generalized method to determine ideal location for placement of any FACT device with a fixed parameter set. Generalized approach is completely based on mathematical model of FACT devices. In Meta-heuristic optimization methods initial population is chosen randomly. Repeatability of optimization results obtained with same initial condition setting is not guaranteed with meta-heuristic methods. According to the proposed method, the objective function is differentiated with respect to the parameter of the corresponding FACT device to be optimized. The basic concept of the generalized method is initially identifying the control parameters of the respective FACT devices and then to determine sensitivity index with FACT device located in each line. Sensitivity index is obtained by differentiating the objective function with respect to device parameters. The parameter that influences power flow in a line is the angle of injected voltage of the series converter. For all three devices considered in this paper, optimal location of the device is mostly influenced by partial derivative of the objective function with respect to angle of injected voltage. The generalized approach cannot applied to the systems where analytical model of the FACTS device is not available.

The proposed method is tested on a 5 Bus and IEEE14 Bus systems for placement of three devices, viz., UPFC, IPFC and OUPFC respectively.

The remaining paper is organized as follows. Section 'Static modeling of FACTS devices' consists of a static modeling of UPFC, IPFC and OUPFC. In Section 'Generalized approach for optimal location of FACTS devices', methods for determining optimal location of UPFC, IPFC and OUPFC are described. Section 'Results' consists of simulation results. Conclusions are presented in Section 'Conclusions'.

Static modeling of FACTS devices

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(P_{ij} and Q_{ij}) from bus i to bus j are obtained from the following equations.

$$P_{ij} = V_i^2 G_{ij} - V_i V_j [G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}] \tag{1}$$

$$Q_{ij} = -V_i^2 (B_{ij} + B_{sh}) - V_i V_j [G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}] \tag{2}$$

where $\delta_{ij} = \delta_i - \delta_j$.

Similarly, P_{ji} and Q_{ji} flow from bus j to bus i are,

$$P_{ji} = V_j^2 G_{ij} - V_j V_i [G_{ij} \cos\delta_{ij} - B_{ji} \sin\delta_{ij}] \tag{3}$$

$$Q_{ji} = -V_j^2 (B_{ij} + B_{sh}) + V_j V_i [G_{ij} \sin\delta_{ij} + B_{ij} \cos\delta_{ij}] \tag{4}$$

Unified power flow controller

The transmission line model with a UPFC connected between bus i and bus j is shown in Fig. 2 and control vector diagram is shown in Fig. 3.

Converter 1 is primarily used to provide real power demand of converter 2 at common DC link terminal from AC power system. Converter 1 can also generate or absorb reactive power at its AC

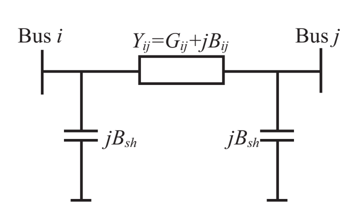


Fig. 1. Transmission line model.

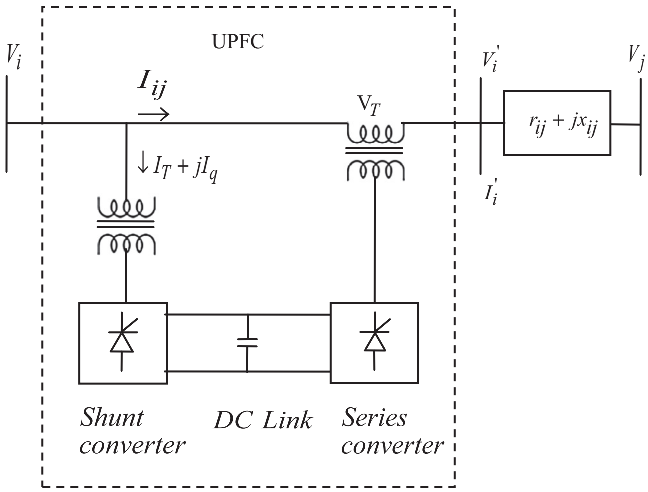
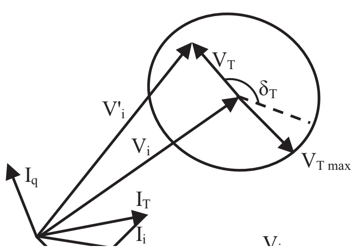


Fig. 2. Basic schematic diagram of the UPFC.



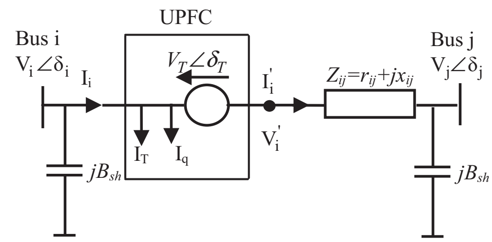


Fig. 4. UPFC equivalent circuit.

$$P^u_{ij} = (V_i^2 + V_i^2)G_{ij} + 2V_iV_TG_{ij}\cos(\delta_T - \delta_i) - V_jV_T[G_{ij}\cos(\delta_T - \delta_j) + B_{ij}\sin(\delta_T - \delta_j)] - V_iV_j[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}] \quad (7)$$

$$Q^u_{ij} = -V_iI_q - V_i^2(B_{ij} + B/2) - V_iV_T[G_{ij}\sin(\delta_T - \delta_i) + B_{ij}\cos(\delta_T - \delta_i)] - V_iV_j[G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}] \quad (8)$$

$$P^u_{ji} = V_j^2G_{ij} - V_iV_j[G_{ij}\cos\delta_{ij} - B_{ij}\sin\delta_{ij}] - V_jV_T[G_{ij}\cos(\delta_T - \delta_j) - B_{ij}\sin(\delta_T - \delta_j)] \quad (9)$$

$$Q^u_{ji} = -V_j^2(B_{ij} + B/2) + V_iV_j[G_{ij}\sin\delta_{ij} + B_{ij}\cos\delta_{ij}] + V_jV_T[G_{ij}\sin(\delta_T - \delta_i) + B_{ij}\cos(\delta_T - \delta_i)] \quad (10)$$

The injected equivalent circuit of Fig. 5 can be obtained using basic circuit theory. The injected active power at bus i (P_{iu}) and bus j (P_{ju}), and reactive powers (Q_{iu} and Q_{ju}) of a line having a UPFC are:

Fig. 3. Vector diagram of UPFC.

$$P_{ju} = V_jV_T[G_{ij}\cos(\delta_T - \delta_j) - B_{ij}\sin(\delta_T - \delta_j)] \quad (12)$$

$$Q_{iu} = V_iI_q + V_iV_T[G_{ij}\sin(\delta_T - \delta_i) + B_{ij}\cos(\delta_T - \delta_i)] \quad (13)$$

$$Q_{ju} = -V_jV_T[G_{ij}\sin(\delta_T - \delta_j) + B_{ij}\cos(\delta_T - \delta_j)] \quad (14)$$

terminal, which is independent of the active power transfer to (or from) DC terminal. Therefore it can also fulfill the function of an independent advanced static VAR compensator providing reactive power compensation for the transmission line and thus executing indirect voltage regulation at the input terminal of the UPFC.

Converter 2 is used to generate a voltage at the fundamental frequency. The inverter output voltage injected in series with line can be used for direct voltage control, series compensation, phase shifter and their combinations. This voltage can internally generate or absorb all the reactive power required by the different type of controls applied and transfers active power at its DC terminal.

Based on the principle of UPFC and the vector diagram, the basic mathematical relations can be given as,

$$V_i' = V_i + V_T,\ Arg(I_q) = Arg(V_i) \pm \pi/2,$$
$$\mathrm{Arg}(I_T) = Arg(V_i),\ I_T^* = \frac{Re[V_TI_i'^*]}{V_i},$$
$$I_q = \frac{\mathrm{Imag}[V_TI_i'^*]}{V_i},\ I_i' = \frac{V_i' - V_j}{Z_{ij}},\ I_{sh} = jV_jB/2 \quad (5)$$

The equivalent circuit of UPFC placed in line-k connected between bus i and bus j is shown in Fig. 4. UPFC has three controllable parameters, namely magnitude, angle of inserted voltage (V_T, δ_T) and magnitude of the current (I_q). The power flow equations from bus i to bus j and vice versa is written as

$$S^u_{ij} = P^u_{ij} + Q^u_{ij} = V_iI^*_{ij} = V_i[I_{sh} + I_i' + (I_T + I_q)]^*$$
$$S^u_{ji} = P^u_{ji} + Q^u_{ji} = V_j(I_{sh} - I_i')^* \quad (6)$$

Active and reactive power flow in the lines having UPFC can be written, using above (5), (6), as

Interline power flow controller

The IPFC comprises a number of Static Synchronous Series Compensators (SSSC) which are solid-state voltage sources converters (VSCs). With this scheme, in addition to providing series reactive compensation, any converter can be controlled to supply real power to the common DC link from its own transmission line. The simplest IPFC consists of two back-to-back DC-to-AC converters, which are connected in series with two transmission lines via transformers. The DC terminals of the converters are connected together via a common DC link. Fig. 6 shows the schematic diagram of such IPFC. IPFC is incorporated between line-l (connecting bus i and bus j) and line-m (connecting bus i and bus k).

A mathematical model for IPFC which will be referred to as power injection model is derived. Based on this, the π-equivalent circuit of IPFC is shown in Fig. 7 and its control vector diagram for line-l (i.e., from bus i to j) is shown in Fig. 8.

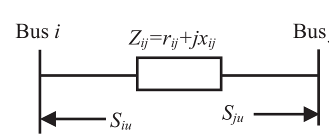


Fig. 5. Injection model of UPFC.

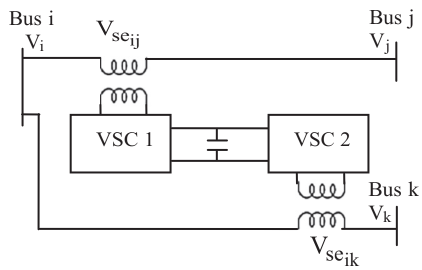


Fig. 6. Schematic diagram of two converter model of IPFC.

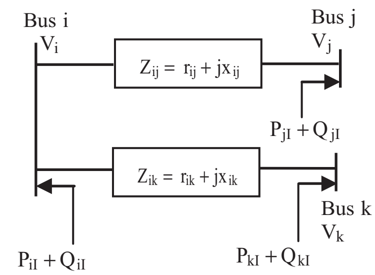


Fig. 9. Injection model of IPFC.

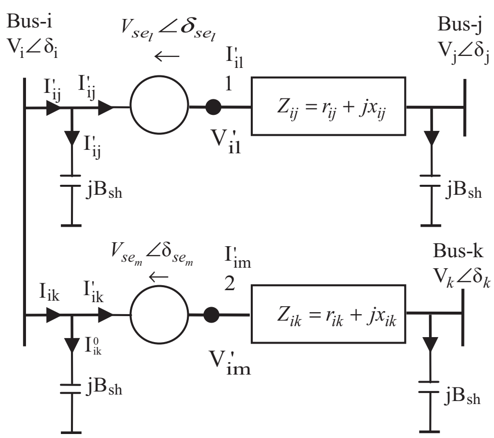


Fig. 7. IPFC equivalent circuit.

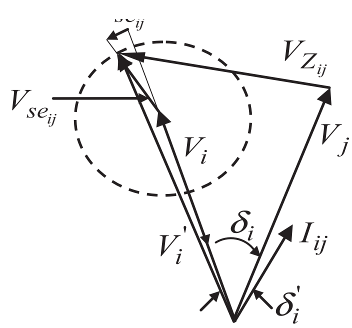


Fig. 8. Vector diagram of IPFC control action for line-l (bus i to j).

In the equivalent circuit, node 1 and 2 are dummy buses which connect the IPFC to the lines l and m respectively and V_i, V_j and V_k are the complex bus voltages at the buses i, j and k respectively, defined as $V_i\angle\theta_i$, $V_j\angle\theta_j$ and $V_k\angle\theta_k$ respectively.

V_{sein} is the complex controllable series Injected voltage source, defined as $V_{sein} = V_{sein}\angle\theta_{sein}$ ($n = j, k$) and Z_{in} ($n = j, k$) is the impedance of the transmission lines and the series coupling transformers, for the sake of simplicity. The active and reactive power injections at buses i, j and k is shown in Fig. 9.

The mathematical derivation is applicable to an IPFC with any number of series converters. From Fig. 7, for lth and mth lines the relation between node voltage (V_j) and current through the line can be derived. Based on the principle of IPFC and the vector diagram, the basic mathematical equations can be given as

$$V'_j = V_i + V_{se_{ij}}, I'_j = \frac{V'_j - V_j}{Z_{ij}},$$
$$I^0_{ij} = jV_jB/2, I_{ij} = I^0_{ij} + I'_{ij} \tag{15}$$

The power flow equations from bus i to bus j and vice versa is written as

$$S^l_{ij} = P^l_{ij} + jQ^l_{ij} = V_iI^*_{ij} = V_i[I^0_{ij} + I'_{ij}]^*$$
$$S^u_{ji} = P^l_{ji} + jQ^l_{ji} = V_jI^*_{ji} = V_j(I^0_{ji} - I'_{ji})^* \tag{16}$$

Active and reactive power flows in the line-l and line-m having IPFC can be written, using above (15), (16), as:

$$P^l_{in} = V_i^2G_{in} - V_iV_n[G_{in}\cos\delta_{in} + B_{in}\sin\delta_{in}] + V_iV_{se_{in}}[G_{in}\cos(\delta_i - \delta_{se_{in}}) + B_{in}\sin(\delta_i - \delta_{se_{in}})] \tag{17}$$

$$Q^l_{in} = -V_i^2(B_{in} + B/2) - V_iV_n[G_{in}\sin\delta_{in} - B_{in}\cos\delta_{in}] + V_iV_{se_{in}}[G_{in}\sin(\delta_i - \delta_{se_{in}}) - B_{in}\cos(\delta_i - \delta_{se_{in}})] \tag{18}$$

$$P^l_{ni} = V_n^2G_{in} - V_iV_n[G_{in}\cos\delta_{in} - B_{in}\sin\delta_{in}]$$

$$- V_nV_{se_{in}}[G_{in}\sin(\delta_n - \delta_{se_{in}}) - B_{in}\cos(\delta_n - \delta_{se_{in}})] \tag{20}$$

According to equivalent circuit of IPFC shown in Fig. 7, power injections of line-l and line-m can be derived as,

$$P_{il} = -V_iV_{se_{in}}[G_{in}\cos\delta_{ise_{in}} + B_{in}\sin\delta_{ise_{in}}] \tag{21}$$

$$P_{nl} = V_nV_{se_{in}}[G_{in}\cos\delta_{nse_{in}} + B_{in}\sin\delta_{nse_{in}}] \tag{22}$$

$$Q_{il} = -V_iV_{se_{in}}[G_{in}\sin\delta_{ise_{in}} - B_{in}\cos\delta_{ise_{in}}] \tag{23}$$

$$Q_{nl} = V_nV_{se_{in}}[G_{in}\sin\delta_{nse_{in}} - B_{in}\cos\delta_{nse_{in}}] \tag{24}$$

Optimal unified power flow controller

The OUPFC is constructed from a PST and a UPFC linked by two triple winding transformers. The PST, which is connected to secondary windings of exciting and injecting transformers, injects a voltage with a fixed phase to the transmission line controlled by mechanical or static switches. The injected voltage changes the transmission angle, depending on system conditions. The UPFC, connected to a tertiary winding of exciting and injecting transformers, consists of two voltage source converters. The back-to-back converters are operated from a common DC link provided by a DC storage capacitor. The basic schematic of the OUPFC is presented in Fig. 10, and its vector diagram is shown in Fig. 11.

The equivalent circuit of OUPFC placed in line-k connected between bus i and bus j is shown in Fig. 12. This model effectively demonstrates OUPFC behavior in steady state within a power system. It has four controllable parameters, namely magnitude, angle of inserted voltage (V_T, δ_T), magnitude of the current (I_q) of the UPFC and phase angle (φ) of the PST.

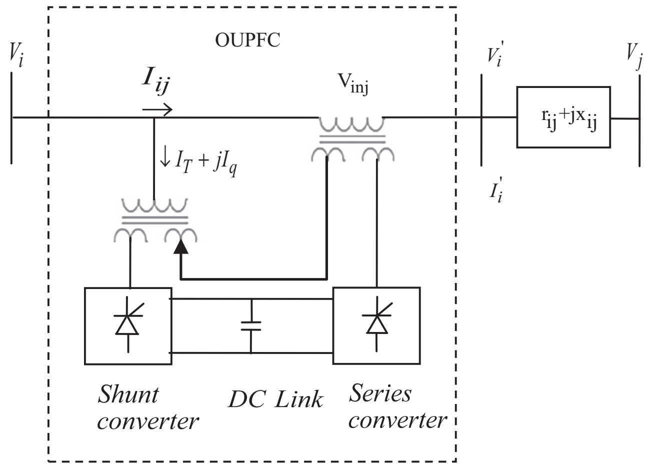


Fig. 10. Pre-phase schematic diagram of OUPFC.

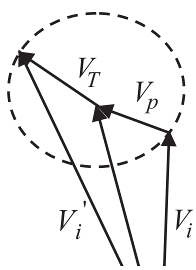


Fig. 11. Vector diagram of OUPFC.

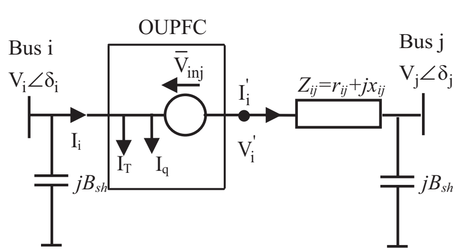


Fig. 12. OUPFC equivalent circuit.

The voltage V_P is injected voltage into the transmission line by PST. The voltage source V_{inj} is the total injected voltage by PST and UPFC ($V_{inj} = V_T + V_P$) and the voltage V'_i is obtained by vectorially adding the total injected voltage V_{inj} to the sending end voltage V_i.

The active and reactive power flow equations of OUPFC connected in line-k can be written as

$$P_{ij}^o = (V_i^2 + V_i^2)G_{ij} + 2V_iV_TG_{ij}\cos(\delta_T - \delta_i) - V_jV_T[G_{ij}\cos(\delta_T - \delta_j) + B_{ij}\sin(\delta_T - \delta_j)] - V_iV_j[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}] + V_i^2T^2G_{ij} - V_iV_jT[G_{ij}\cos(\delta_{ij} + \varphi) + B_{ij}\sin(\delta_{ij} + \varphi)] \quad (25)$$

$$Q_{ij}^o = -V_iI_q - V_i^2(B_{ij} + B/2 + T^2B_{ij}) - V_iV_j[G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}] - V_iV_T[G_{ij}\sin(\delta_T - \delta_i) + B_{ij}\cos(\delta_T - \delta_i)] - V_iV_jT[G_{ij}\sin(\delta_{ij} + \varphi) - B_{ij}\cos(\delta_{ij} + \varphi)] \quad (26)$$

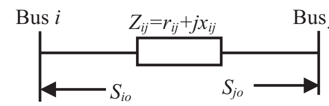


Fig. 13. Injection model of OUPFC.

$$P_{ji}^o = 2V_j^2G_{ij} - V_iV_j[G_{ij}\cos\delta_{ij} - B_{ij}\sin\delta_{ij}] - V_jV_T[G_{ij}\cos(\delta_T - \delta_j) - B_{ij}\sin(\delta_T - \delta_j)] - V_jV_iT[G_{ij}\cos(\delta_{ij} + \varphi) - B_{ij}\sin(\delta_{ij} + \varphi)] \quad (27)$$

$$Q_{ji}^o = -V_j^2(B_{ij} + B/2) + V_iV_j[G_{ij}\sin\delta_{ij} + B_{ij}\cos\delta_{ij}] + V_jV_T[G_{ij}\sin(\delta_T - \delta_i) + B_{ij}\cos(\delta_T - \delta_i)] - V_j^2B_{ij} + V_jV_iT[G_{ij}\sin(\delta_{ij} + \varphi) + B_{ij}\cos(\delta_{ij} + \varphi)] \quad (28)$$

where T = secφ.

The injected model of OUPFC of Fig. 13 can be obtained using basic circuit theory. The injected active power at bus i (P_{io}) and bus j (P_{jo}), and reactive powers (Q_{io} and Q_{jo}) of a line having a OUPFC are:

$$P_{io} = -V_i^2K^2G_{ij} - V_iV_jK[G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}] - V_T^2G_{ij} - 2V_iV_TG_{ij}\cos(\delta_T - \delta_i) + V_jV_T[G_{ij}\cos(\delta_T - \delta_j) + B_{ij}\sin(\delta_T - \delta_j)] \quad (29)$$

$$P_{jo} = -V_iV_jK[G_{ij}\sin\delta_{ij} + B_{ij}\cos\delta_{ij}] + V_jV_T[G_{ij}\cos(\delta_T - \delta_j) - B_{ij}\sin(\delta_T - \delta_j)] \quad (30)$$

$$Q_{io} = V_i^2K^2B_{ij} + V_iV_jK[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}]$$

$$+ B_{ij}\cos(\delta_T - \delta_j)] \quad (32)$$

where $K = \tan(\phi)$.

Generalized approach for optimal location of FACTS devices

Determination of optimal location for placement of the FACTS devices in earlier studies was decided based on improvement in stability and damping of oscillations in dynamic state. In this work, the following objectives have been considered in steady state.

(i) Reduction in total system real power loss (P_{LT}).
(ii) Reduction in real power flow performance index (PI).

While placing a FACTS device to reduce the real power loss in a particular line as suggested in [1] may increase the total system loss and/or may increase overloading elsewhere. In some cases, unwanted loop flows will be eliminated which results in reduction of total active power loss [1].

Method 1: Total system loss sensitivity indices

The exact loss formula of N-bus system is, from [41],

$$P'_{LT} = \sum_{j=1}^{N}\sum_{k=1}^{N}[\alpha_{jk}(P_jP_k + Q_jQ_k) + \beta_{jk}(Q_jP_k - P_jQ_k)] \quad (33)$$

where P_j and Q_j are real and reactive powers injected at bus j and the loss coefficients α and β are defined by

$$a_{jk} = \frac{r_{jk}}{V_jV_k}\cos(\delta_j - \delta_k) \text{ and } \beta_{jk} = \frac{r_{jk}}{V_jV_k}\sin(\delta_j - \delta_k) \quad (34)$$

where r_{jk} is the real part of the jth row and kth column element of Z-bus, matrix. The total system loss after placing the FACT devices can be written as:

$$P_{LT} = \begin{cases} P'_{LT} - (P_{iu} + P_{ju}) & \text{for UPFC} \\ P'_{LT} - (P_{il} + P_{jl}) & \text{for IPFC} \\ P'_{LT} - (P_{io} + P_{jo}) & \text{for OUPFC} \end{cases} \tag{35}$$

In the case of UPFC the sensitivity factors, which are obtained as partial derivatives of total system real power loss with respect to the parameters of UPFC placed in line-k can be defined as

$$b_1^u = \left.\frac{\partial P_{LT}}{\partial V_T}\right|_{V_T=0} = \text{Total loss sensitivity with respect to } V_T$$

$$b_2^u = \left.\frac{\partial P_{LT}}{V_T\partial \delta_T}\right|_{\delta_T=0} = \text{Total loss sensitivity with respect to } \delta_T$$

$$b_3^u = \left.\frac{\partial P_{LT}}{\partial I_q}\right|_{I_q=0} = \text{Total loss sensitivity with respect to } I_q$$

where $k = 1, 2 \ldots N_l$ and N_l is total number of lines.

The sensitivity factors $b_1{}^u$, $b_2{}^u$ and $b_3{}^u$ are computed using (35) at a base case load flow. Consider a line-k connected between bus i and bus j.

$$b_1^u = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i \partial V_T}\right|_{V_T=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j \partial V_T}\right|_{V_T=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i \partial V_T}\right|_{V_T=0}$$

$$b_2^u = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i V_T\partial \delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j V_T\partial \delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i V_T\partial \delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j V_T\partial \delta_T}\right|_{\delta_T=0} - \left.\frac{1}{V_T}\left(\frac{\partial P_{iu}}{\partial \delta_T} + \frac{\partial P_{ju}}{\partial \delta_T}\right)\right|_{\delta_T=0} \tag{37}$$

$$b_3^u = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i \partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j \partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i \partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j \partial I_q}\right|_{I_q=0} - \left.\left(\frac{\partial P_{iu}}{\partial I_q} + \frac{\partial P_{ju}}{\partial I_q}\right)\right|_{I_q=0} \tag{38}$$

where

$$\frac{\partial P_{LT}}{\partial P_i} = 2\sum_{m=1}^{N}(\alpha_{im}P_m - \beta_{im}Q_m) \text{ and } \frac{\partial P_{LT}}{\partial Q_i} = 2\sum_{m=1}^{N}(\alpha_{im}Q_m + \beta_{im}P_m) \tag{39}$$

The terms, $\left.\frac{\partial P_i}{\partial V_T}\right|_{V_T=0}$, $\left.\frac{\partial P_j}{\partial V_T}\right|_{V_T=0}$, $\left.\frac{\partial P_i}{V_T\partial \delta_T}\right|_{\delta_T=0}$, $\left.\frac{\partial P_j}{V_T\partial \delta_T}\right|_{\delta_T=0}$, $\left.\frac{\partial P_i}{\partial I_q}\right|_{I_q=0}$ and $\left.\frac{\partial P_j}{\partial I_q}\right|_{I_q=0}$ can be obtained using (11) and (12) respectively and are given below:

$$\left.\frac{\partial P_i}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial P_{iu}}{\partial V_T}\right|_{V_T=0} = -2V_iG_{ij}\cos\delta_i + V_j[G_{ij}\cos\delta_j - B_{ij}\sin\delta_j] \tag{40}$$

$$\left.\frac{\partial P_j}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial P_{ju}}{\partial V_T}\right|_{V_T=0} = V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \tag{41}$$

$$\left.\frac{\partial P_i}{V_T\partial \delta_T}\right|_{\delta_T=0} = \left.\frac{\partial P_{iu}}{V_T\partial \delta_T}\right|_{\delta_T=0} = -2V_iG_{ij}\sin\delta_i + V_j[G_{ij}\sin\delta_j + B_{ij}\cos\delta_j] \tag{42}$$

$$\left.\frac{\partial P_j}{V_T\partial \delta_T}\right|_{\delta_T=0} = \left.\frac{\partial P_{ju}}{V_T\partial \delta_T}\right|_{\delta_T=0} = V_j[G_{ij}\sin\delta_j - B_{ij}\cos\delta_j] \tag{43}$$

$$\left.\frac{\partial P_i}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial P_{iu}}{\partial I_q}\right|_{I_q=0} = 0 \tag{44}$$

$$\left.\frac{\partial P_j}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial P_{ju}}{\partial I_q}\right|_{I_q=0} = 0 \tag{45}$$

Using (13) and (14), the derivative of the reactive power injections with respect to FACTS parameters can be derived as

$$\left.\frac{\partial Q_i}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial Q_{iu}}{\partial V_T}\right|_{V_T=0} = V_i[-G_{ij}\sin\delta_i + B_{ij}\cos\delta_i] \tag{46}$$

$$\left.\frac{\partial Q_j}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial Q_{ju}}{\partial V_T}\right|_{V_T=0} = -V_i[-G_{ij}\sin\delta_j + B_{ij}\cos\delta_j] \tag{47}$$

$$\left.\frac{\partial Q_i}{V_T\partial \delta_T}\right|_{\delta_T=0} = \left.\frac{\partial Q_{iu}}{V_T\partial \delta_T}\right|_{\delta_T=0} = V_j[G_{ij}\cos\delta_i + B_{ij}\sin\delta_i] \tag{48}$$

$$\left.\frac{\partial Q_j}{V_T\partial \delta_T}\right|_{\delta_T=0} = \left.\frac{\partial Q_{ju}}{V_T\partial \delta_T}\right|_{\delta_T=0} = -V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \tag{49}$$

$$\left.\frac{\partial Q_i}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial Q_{iu}}{\partial I_q}\right|_{I_q=0} = V_i \tag{50}$$

$$\left.\frac{\partial Q_j}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial Q_{ju}}{\partial I_q}\right|_{I_q=0} = 0 \tag{51}$$

obtained by partially differentiating total power loss with respect to injected voltage V_{se} and phase angle δ_{se} in line-l, appears to be same as $b_1{}^u$ and $b_2{}^u$. In the case of IPFC, term similar to $b_3{}^u$ will be absent as there is no shunt current injection. They can be defined as

$$b_1^I = \left.\frac{\partial P_{LT}}{\partial V_{se}}\right|_{V_{se}=0} = \text{Total Loss Sensitivity w.r.t to } V_{se}$$

$$b_2^I = \left.\frac{\partial P_{LT}}{V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0} = \text{Total Loss Sensitivity w.r.t to } \delta_{se}$$

$$b_1^I = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i \partial V_{se}}\right|_{V_{se}=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j \partial V_{se}}\right|_{V_{se}=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i \partial V_{se}}\right|_{V_{se}=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j \partial V_{se}}\right|_{V_{se}=0} - \left.\left(\frac{\partial P_{il}}{\partial V_{se}} + \frac{\partial P_{jl}}{\partial V_{se}}\right)\right|_{V_{se}=0} \tag{52}$$

$$b_2^I = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0} - \left.\left(\frac{\partial P_{il}}{\partial \delta_{se}} + \frac{\partial P_{jl}}{\partial \delta_{se}}\right)\right|_{\delta_{se}=0} \tag{53}$$

where $k = 1, 2 \ldots N_l$ and N_l is total number of lines.

The sensitivity factors $b_1{}^I$ and $b_2{}^I$ are computed using (35) at a base case load flow. Consider a line-k connected between bus i and bus j.

The terms, $\left.\frac{\partial P_i}{\partial V_{se}}\right|_{V_{se}=0}$, $\left.\frac{\partial P_j}{\partial V_{se}}\right|_{V_{se}=0}$, $\left.\frac{\partial P_i}{V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0}$ and $\left.\frac{\partial P_j}{V_{se}\partial \delta_{se}}\right|_{\delta_{se}=0}$ can be obtained using (21) and (22) respectively and are given below:

$$\left.\frac{\partial P_i}{\partial V_{se}}\right|_{V_{se}=0} = \left.\frac{\partial P_{il}}{\partial V_{se}}\right|_{V_{se}=0} = -V_i[G_{ij}\cos\delta_i + B_{ij}\sin\delta_i] \tag{54}$$

$$\left.\frac{\partial P_j}{\partial V_{se}}\right|_{V_{se}=0} = \left.\frac{\partial P_{jI}}{\partial V_{se}}\right|_{Vse=0} = V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \quad (55)$$

$$\left.\frac{\partial P_i}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = \left.\frac{\partial P_{iI}}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = -V_i[G_{ij}\sin\delta_i - B_{ij}\cos\delta_i] \quad (56)$$

$$\left.\frac{\partial P_j}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = \left.\frac{\partial P_{jI}}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = V_j[G_{ij}\sin\delta_j - B_{ij}\cos\delta_j] \quad (57)$$

Using (23) and (24), the derivative of the reactive power injections with respect to FACTS parameters can be derived as

$$\left.\frac{\partial Q_i}{\partial V_{se}}\right|_{V_{se}=0} = \left.\frac{\partial Q_{iI}}{\partial V_{se}}\right|_{V_{se}=0} = -V_i[G_{ij}\sin\delta_i - B_{ij}\cos\delta_i] \quad (58)$$

$$\left.\frac{\partial Q_j}{\partial V_{se}}\right|_{V_{se}=0} = \left.\frac{\partial Q_{jI}}{\partial V_{se}}\right|_{V_{se}=0} = V_j[G_{ij}\sin\delta_j - B_{ij}\cos\delta_j] \quad (59)$$

$$\left.\frac{\partial Q_i}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = \left.\frac{\partial Q_{iI}}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = V_i[G_{ij}\cos\delta_i + B_{ij}\sin\delta_i] \quad (60)$$

$$\left.\frac{\partial Q_j}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = \left.\frac{\partial Q_{jI}}{V_{se}\partial\delta_{se}}\right|_{\delta_{se}=0} = -V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \quad (61)$$

The factors $b_1{}^I$ and $b_2{}^I$ are found by substituting (54)–(61) in (52) and (53), respectively.

In the case of OUPFC the sensitivity factors ($b_1{}^o$, $b_2{}^o$, $b_3{}^o$ and $b_4{}^o$), are obtained by partially differentiating total power loss with respect to Magnitude of injected Voltage V_T, Phase angle of injected voltage δ_T, magnitude of shunt current I_q and phase angle of Phase

$$\left.\partial V_T\right|_{V_T=0}$$

$$b_2^o = \left.\frac{\partial P_{LT}}{V_T\partial\delta_T}\right|_{\delta_T=0} = \text{Total loss sensitivity with respect to } \delta_T$$

$$b_3^o = \left.\frac{\partial P_{LT}}{\partial I_q}\right|_{I_q=0} = \text{Total loss sensitivity with respect to } I_q$$

$$b_4^o = \left.\frac{\partial P_{LT}}{\partial\varphi}\right|_{\varphi=0} = \text{Total loss sensitivity with respect to } \varphi$$

where $k = 1, 2\ldots N_l$ and N_1 is total number of lines.

The sensitivity factors $b_1{}^o$, $b_2{}^o$, $b_3{}^o$ and $b_4{}^o$ are computed using (35) at a base case load flow. Consider a line-k connected between bus i and bus j.

$$b_1^o = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i\partial V_T}\right|_{V_T=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j\partial V_T}\right|_{V_T=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i\partial V_T}\right|_{V_T=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j\partial V_T}\right|_{V_T=0} - \left.\left(\frac{\partial P_{io}}{\partial V_T} + \left.\frac{\partial P_{jo}}{\partial V_T}\right|\right)\right|_{V_T=0} \quad (62)$$

$$b_2^o = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i V_T\partial\delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j V_T\partial\delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i V_T\partial\delta_T}\right|_{\delta_T=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j V_T\partial\delta_T}\right|_{\delta_T=0} - \frac{1}{V_T}\left.\left(\frac{\partial P_{io}}{\partial\delta_T} + \left.\frac{\partial P_{jo}}{\partial\delta_T}\right|\right)\right|_{\delta_T=0} \quad (63)$$

$$b_3^o = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i\partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j\partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i\partial I_q}\right|_{I_q=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j\partial I_q}\right|_{I_q=0} - \left.\left(\frac{\partial P_{io}}{\partial I_q} + \left.\frac{\partial P_{jo}}{\partial I_q}\right|\right)\right|_{I_q=0} \quad (64)$$

$$b_4^o = \left.\frac{\partial P_{LT}\partial P_i}{\partial P_i\partial\varphi_k}\right|_{\varphi_k=0} + \left.\frac{\partial P_{LT}\partial P_j}{\partial P_j\partial\varphi_k}\right|_{\varphi_k=0} + \left.\frac{\partial P_{LT}\partial Q_i}{\partial Q_i\partial\varphi_k}\right|_{\varphi_k=0} + \left.\frac{\partial P_{LT}\partial Q_j}{\partial Q_j\partial\varphi_k}\right|_{\varphi_k=0} - \left.\left(\frac{\partial P_{io}}{\partial\varphi_k} + \left.\frac{\partial P_{jo}}{\partial\varphi_k}\right|\right)\right|_{\varphi_k=0} \quad (65)$$

The terms, $\left.\frac{\partial P_i}{\partial\varphi}\right|_{\varphi=0}$, $\left.\frac{\partial P_j}{\partial\varphi}\right|_{\varphi=0}$ $\left.\frac{\partial P_i}{\partial V_T}\right|_{V_T=0}$, $\left.\frac{\partial P_j}{\partial V_T}\right|_{V_T=0}$, $\left.\frac{\partial P_i}{V_T\partial\delta_T}\right|_{\delta_T=0}$, $\left.\frac{\partial P_j}{V_T\partial\delta_T}\right|_{\delta_T=0}$, $\left.\frac{\partial P_i}{\partial I_q}\right|_{I_q=0}$ and $\left.\frac{\partial P_j}{\partial I_q}\right|_{I_q=0}$ can be obtained using (29) and (30) respectively and are given below.

$$\left.\frac{\partial P_i}{\partial\varphi}\right|_{\varphi=0} = \left.\frac{\partial P_{io}}{\partial\varphi}\right|_{\varphi=0} = -V_iV_j(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \quad (66)$$

$$\left.\frac{\partial P_j}{\partial\varphi}\right|_{\varphi=0} = \left.\frac{\partial P_{jo}}{\partial\varphi}\right|_{\varphi=0} = -V_iV_j(G_{ij}\sin\delta_{ij} + B_{ij}\cos\delta_{ij}) \quad (67)$$

$$\left.\frac{\partial P_i}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial P_{io}}{\partial V_T}\right|_{V_T=0} = -2V_iG_{ij}\cos\delta_i + V_j[G_{ij}\cos\delta_j - B_{ij}\sin\delta_j] \quad (68)$$

$$\left.\frac{\partial P_j}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial P_{jo}}{\partial V_T}\right|_{V_T=0} = V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \quad (69)$$

$$\left.\frac{\partial P_i}{V_T\partial\delta_T}\right|_{\delta_T=0} = \left.\frac{\partial P_{io}}{V_T\partial\delta_T}\right|_{\delta_T=0} = -2V_iG_{ij}\sin\delta_i + V_j[G_{ij}\sin\delta_j + B_{ij}\cos\delta_j] \quad (70)$$

$$\partial P_i \quad \partial P_{io}$$

$$\left.\partial I_q\right|_{I_q=0} \quad \left.\partial I_q\right|_{I_q=0}$$

$$\left.\frac{\partial P_j}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial P_{jo}}{\partial I_q}\right|_{I_q=0} = 0 \quad (73)$$

Using (31) and (32), the derivative of the reactive power injections with respect to FACTS parameters can be derived as

$$\left.\frac{\partial Q_i}{\partial\varphi}\right|_{\varphi=0} = \left.\frac{\partial Q_{io}}{\partial\varphi}\right|_{\varphi=0} = V_iV_j[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}] \quad (74)$$

$$\left.\frac{\partial Q_j}{\partial\varphi}\right|_{\varphi=0} = \left.\frac{\partial Q_{jo}}{\partial\varphi}\right|_{\varphi=0} = -V_iV_j[G_{ij}\cos\delta_{ij} - B_{ij}\sin\delta_{ij}] \quad (75)$$

$$\left.\frac{\partial Q_i}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial Q_{io}}{\partial V_T}\right|_{V_T=0} = V_i[-G_{ij}\sin\delta_i + B_{ij}\cos\delta_i] \quad (76)$$

$$\left.\frac{\partial Q_j}{\partial V_T}\right|_{V_T=0} = \left.\frac{\partial Q_{jo}}{\partial V_T}\right|_{V_T=0} = -V_i[-G_{ij}\sin\delta_j + B_{ij}\cos\delta_j] \quad (77)$$

$$\left.\frac{\partial Q_i}{V_T\partial\delta_T}\right|_{\delta_T=0} = \left.\frac{\partial Q_{io}}{V_T\partial\delta_T}\right|_{\delta_T=0} = V_j[G_{ij}\cos\delta_i + B_{ij}\sin\delta_i] \quad (78)$$

$$\left.\frac{\partial Q_j}{V_T\partial\delta_T}\right|_{\delta_T=0} = \left.\frac{\partial Q_{jo}}{V_T\partial\delta_T}\right|_{\delta_T=0} = -V_j[G_{ij}\cos\delta_j + B_{ij}\sin\delta_j] \quad (79)$$

$$\left.\frac{\partial Q_i}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial Q_{io}}{\partial I_q}\right|_{I_q=0} = V_i \quad (80)$$

$$\left.\frac{\partial Q_j}{\partial I_q}\right|_{I_q=0} = \left.\frac{\partial Q_{jo}}{\partial I_q}\right|_{I_q=0} = 0 \quad (81)$$

The factors b_1^o, b_2^o, b_3^o and b_4^o are found by substituting (66)–(81) in (62) and (65), respectively.

Method 2: Real power flow PI sensitivity indices

The Real power line flow Performance Index (PI) [1] described by the following equation indicates the severity of the system loading under normal and contingency conditions.

$$PI = \sum_{m=1}^{N_1} \frac{W_m}{2n} \left(\frac{P_{1m}}{P_{1m}^{\max}} \right)^{2n} \tag{82}$$

where W_m is a real non-negative weight coefficient, P_{1m} is the real power flow and $P_{1m}^{\max}$ is the rated capacity of line-m. The exponent n reflects the importance of lines. As long as the lines are not overloaded, PI will be small and as the lines get over loaded, PI reach a high value. PI acts as a measure of severity of overloading of lines in a given power system. Second order performance indices are used for contingency selection algorithms. PI will be small for light to normal loads and will be high for overloaded lines. However they suffer from masking effects.

From the previous studies it is learnt that the system with one large violation is more severe than that with several small violations. Due to masking effect, it becomes difficult to discriminate between single large violations and several small violations of line overloading. Masking effect can be avoided by taking higher order derivative of performance indices ($n > 1$). In this work, the value of exponent has been taken as 2 and $W_m = 1.0$.

In the case of UPFC, sensitivity factors $c_1{}^u$, $c_2{}^u$ and $c_3{}^u$ can be defined as partial derivative of PI with respect to magnitude of

$$c_1^u = \frac{\partial PI}{\partial V_T}\bigg|_{V_T=0} = \text{PI Sensitivity with respect to } V_T,$$

$$c_2^u = \frac{\partial PI}{V_T \partial \delta_T}\bigg|_{\delta_T=0} = \text{PI Sensitivity with respect to } \delta_T,$$

$$c_3^u = \frac{\partial PI}{\partial I_q}\bigg|_{I_q=0} = \text{PI Sensitivity with respect to } I_q,$$

Using (82), sensitivity of PI with respect to UPFC parameter X_k (V_T, δ_T and I_q), with respect to IPFC parameter X_k (V_{se} and δ_{se}) and with respect to OUPFC parameter X_k (V_T, δ_T, I_q and φ), connected between bus i and bus j can be given as

$$\frac{\partial PI}{\partial X_k} = \sum_{m=1}^{N_1} W_m P_{1m}^3 \left(\frac{1}{P_{1m}^{\max}} \right)^4 \frac{\partial P_{1m}}{\partial X_k} \tag{83}$$

Using the DC load flow equation [42], real power flowing through line-m (P_{1m}) can be represented as sum of real power injections. Assume s be the slack bus.

$$P_{1m} = \begin{cases} \sum_{n=1, n\neq s}^{N} S_{mn} P_n & \text{for } m \neq k \\ \sum_{n=1, n\neq s}^{N} S_{mn} P_n + P_j & \text{for } m \neq k \end{cases} \tag{84}$$

where N is number of buses in the system and S_{mn} is the mnth element of matrix $[S_f]$ which relates line power flow with power injections [42] at the buses without FACTS devices. It is found in line k (between bus i and bus j) there is an additional flow of P_j at bus j when a FACT device is connected, as shown in Fig. 2.

Using (83) and (84) the following relationship can be derived,

$$\frac{\partial P_{1m}}{\partial X_k} = \begin{cases} \left(S_{mi} \frac{\partial P_i}{\partial X_k} + S_{mj} \frac{\partial P_j}{\partial X_k} \right) & \text{for } m \neq k \\ \left(S_{mi} \frac{\partial P_i}{\partial X_k} + S_{mj} \frac{\partial P_j}{\partial X_k} \right) + \frac{\partial P_j}{\partial X_k} & \text{for } m = k \end{cases} \tag{85}$$

The terms, $\frac{\partial P_i}{\partial V_T}\big|_{V_T=0}$, $\frac{\partial P_j}{\partial V_T}\big|_{V_T=0}$, $\frac{\partial P_i}{V_T \partial \delta_T}\big|_{\delta_T=0}$, $\frac{\partial P_j}{V_T \partial \delta_T}\big|_{\delta_T=0}$, $\frac{\partial P_i}{\partial I_q}\big|_{I_q=0}$ and $\frac{\partial P_j}{\partial I_q}\big|_{I_q=0}$ can be derived using (40)–(45) respectively. The sensitivity factors $c_1{}^u$, $c_2{}^u$ and $c_3{}^u$ can be derived by partially differentiating PI w.r.t V_T, δ_T and I_q in line-k respectively.

Sensitivity index is calculated with respect to the voltage magnitude and phase angle of the series converter of the UPFC.

In the case of IPFC the sensitivity factors '$c_1{}^I$ and $c_2{}^I$' which are obtained by partially differentiating real power flow performance index with respect to injected voltage V_{se} and phase angle ϕ_{se} in line-l, which are same as $c_1{}^u$ and $c_2{}^u$ respectively. In the case of IPFC, term similar to $c_3{}^u$ will be absent as there is no shunt current injection.

$$c_1^I = \frac{\partial PI}{\partial V_{se}}\bigg|_{V_{se}=0} = \text{PI Sensitivity with respect to } V_{se}$$

$$c_2^I = \frac{\partial PI}{V_{se} \partial \delta_{se}}\bigg|_{\delta_{se}=0} = \text{PI Sensitivity with respect to } \delta_{se}$$

Using (40), Sensitivity of PI with respect to IPFC parameter X_k (V_{se} and δ_{se}) and the terms, $\frac{\partial P_i}{\partial V_{se}}\big|_{V_{se}=0}$, $\frac{\partial P_j}{\partial V_{se}}\big|_{V_{se}=0}$, $\frac{\partial P_i}{V_{se} \partial \delta_{se}}\big|_{\delta_{se}=0}$, and $\frac{\partial P_j}{V_{se} \partial \delta_{se}}\big|_{\delta_{se}=0}$, can be derived using (54)–(57) respectively. The sensitivity factors $c_1{}^I$ and $c_2{}^I$ can be derived by partially differentiating

I_q and Phase angle of phase shifting transformer ϕ. These are represented by $c_1{}^o$, $c_2{}^o$, $c_3{}^o$ and $c_4{}^o$. Except the last parameter, all are similar for UPFC.

$$c_1^o = \frac{\partial PI}{\partial V_T}\bigg|_{V_T=0} = \text{PI Sensitivity with respect to } V_T,$$

$$c_2^o = \frac{\partial PI}{V_T \partial \delta_T}\bigg|_{\delta_T=0} = \text{PI Sensitivity with respect to } \delta_T,$$

$$c_3^o = \frac{\partial PI}{\partial I_q}\bigg|_{I_q=0} = \text{PI Sensitivity with respect to } I_q,$$

$$c_4^o = \frac{\partial PI}{\partial \varphi}\bigg|_{\varphi=0} = \text{PI Sensitivity with respect to } \varphi,$$

Using (40), the sensitivity of PI with respect to OUPFC parameter X_k (φ, V_T, δ_T and I_q), the terms, $\frac{\partial P_i}{\partial \varphi}\big|_{\varphi=0}$, $\frac{\partial P_j}{\partial \varphi}\big|_{\varphi=0}$, $\frac{\partial P_i}{\partial V_T}\big|_{V_T=0}$, $\frac{\partial P_j}{\partial V_T}\big|_{V_T=0}$, $\frac{\partial P_i}{V_T \partial \delta_T}\big|_{\delta_T=0}$, $\frac{\partial P_j}{V_T \partial \delta_T}\big|_{\delta_T=0}$, $\frac{\partial P_i}{\partial I_q}\big|_{I_q=0}$ and $\frac{\partial P_j}{\partial I_q}\big|_{I_q=0}$ can be derived using (66)–(73) respectively. The sensitivity factors $c_1{}^o$, $c_2{}^o$, $c_3{}^o$ and $c_4{}^o$ can be derived by partially differentiating PI w.r.t V_T, δ_T, I_q and φ in line k respectively.

Criteria for optimal placement

The criteria followed for determination of optimal placement of any FACTS device (UPFC, IPFC and OUPFC) are as follows:

(i) The device is placed in a line-k which has the least sensitivity with respect to magnitude of inserted voltage and current.

(ii) The UPFC is placed in a line-*k* which has the largest absolute value of the sensitivity with respect to phase angle of inserted voltage.

(iii) The FACTS device should not be placed in the line containing generation buses, even if the sensitivity is the highest these.

Results

To determine optimal location for UPFC, the objective functions are differentiated with respect to magnitude of inserted voltage, angle of inserted voltage and magnitude of current ($b_1{}^u$, $b_2{}^u$, $b_3{}^u$ and $c_1{}^u$, $c_2{}^u$, $c_3{}^u$). To determine optimal location for IPFC, the objective functions are differentiated with respect to magnitude of series injected voltage of master line, angle of injected voltage for master line ($b_1{}^I$, $b_2{}^I$ and $c_1{}^I$, $c_2{}^I$). To determine optimal location for OUPFC, the objective functions are differentiated with respect to angle of the Phase shifting transformer and, voltage magnitude of inserted voltage, angle of inserted voltage and magnitude of current ($b_1{}^o$, $b_2{}^o$, $b_3{}^o$, $b_4{}^o$ and $c_1{}^o$, $c_2{}^o$, $c_3{}^o$, $c_4{}^o$). Broadly two sets of indices are obtained for every device-total system loss sensitivity indices denoted by (b) and real power flow performance indices denoted by (c). Proper location of these devices in the network is very important due to their high cost. The effectiveness of the proposed methods is demonstrated on 5 Bus and IEEE 14 Bus Systems.

5-bus system

Five-bus system [43] shown in Fig. 14 consists of three generator buses, two load buses and six transmission lines. The lines 1–2

bus 5 is considered as reference bus.

Sensitivities are calculated for each control parameter of UPFC, IPFC and OUPFC associated with every line one at a time for the same operating conditions. The sensitivities of total system real power loss (method-1) and real power flow PI (method-2) with

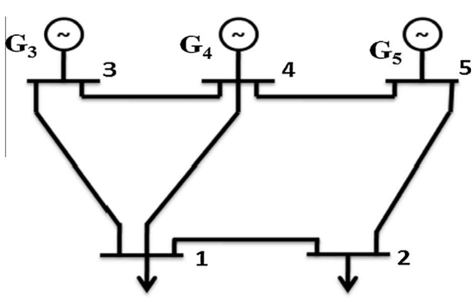


Fig. 14. Five bus system.

Table 1
Sensitivities of 5-bus system using different methods.

Line-*k*		UPFC					
		Method-1			Method-2		
No	*i–j*	$b_1{}^u$	$b_2{}^u$	$b_3{}^u$	$c_1{}^u$	$c_2{}^u$	$c_3{}^u$
1	1–2	5.8680	**10.7532**	0.0226	**−0.6257**	−2.3363	0.0
2	1–3	4.3867	9.4317	0.0226	−0.0357	−1.0725	0.0
3	1–4	5.1716	7.4567	0.0226	0.9474	**4.0746**	0.0
4	2–5	−0.6102	−1.1308	−0.0053	−0.4095	−2.4411	0.0
5	3–4	**−2.1869**	−5.7992	**−0.0215**	−0.2321	−1.2537	0.0
6	4–5	−0.8189	0.9053	0.0066	0.5042	2.5421	0.0

Table 2
Sensitivities of 5-bus system.

Line-*k*		Power flows	IPFC			
			Method-1		Method-2	
No	*i–j*	Base case	$b_1{}^I$	$b_2{}^I$	$c_1{}^I$	$c_2{}^I$
1	1–2	0.9634	6.7803	10.9414	−0.5376	−2.3181
2	1–3	−6.9911	−2.0318	11.5729	−0.6555	−0.8657
3	1–4	**−8.9723**	−2.9095	**12.9204**	0.1670	**4.6022**
4	2–5	−4.0387	**−4.7011**	2.0973	**−0.6590**	−2.2442
5	3–4	2.7875	0.5771	−5.1127	−0.3112	−1.2733
6	4–5	1.0561	0.1438	1.1078	0.4536	2.5315

respect to UPFC, IPFC and OUPFC control parameters are presented in Tables 1–3 respectively

Partial derivative of the objective function with respect to two parameters should be negative maximum for finding the optimal location of any of the FACT devices. The two parameters are voltage magnitude and the current injected. Hence, to highest negative sensitivities in $b_1{}^u$ and $b_3{}^u$ in the case of UPFC and $b_1{}^I$ in the case of IPFC and $b_1{}^o$, $b_3{}^o$ in the case of OUPFC are optimal locations for placement of the corresponding device with respect to magnitude of voltage or current. There is only one index available in the case of IPFC, as it does not have any current injection. The highest absolute value of partial derivative of objective function with respect to phase angle of voltage will be the best location for placement of FACT device. The highest absolute values of sensitivities in $b_2{}^u$ in the case of UPFC and in $b_2{}^I$ in the case of IPFC and in $b_2{}^o$, $b_4{}^o$ in the case of OUPFC are potential locations for placement of FACT

flow.

Table 1 (method-1) presents the sensitivity factors of UPFC placed in every line ($b_1{}^u$, $b_2{}^u$ and $b_3{}^u$). From Table 1, column 4, the sensitivity with respect to voltage magnitude, $b_1{}^u$ in line-5 is the least. This indicates that maximum reduction in total system real power loss is obtained, if UPFC is placed in line-5, it would result in. The sensitivity of total system real power loss with respect to phase angle ($b_2{}^u$) of UPFC placed in line-1 is the highest followed by line-2. This indicates that placement of UPFC in line-1 with negative phase shift will reduce the total system real power loss more than the placement in other lines. The sensitivity factor $b_3{}^u$ is almost same for each line, which is an indication of uniform voltage profile throughout the system. The sensitivity for lines-4 and 5 are more negative than others.

Table 2 (method-1) presents the sensitivity factors $b_1{}^u$ and $b_2{}^u$ of IPFC placed in every line in turns. From Table 2, column 4, the sensitivity factors $b_1{}^I$ are small, which indicates that reduction in total system loss will be less. For voltage magnitude control, line-4 is suitable as its sensitivity is the least. The absolute value of sensitivity with respect to phase angle ($b_2{}^I$) of IPFC placed in line-3 is the highest. This indicates that placement of IPFC in line-3 will reduce the total system real power loss which is a positive value. This indicates that placement of IPFC in line-3 with negative phase shift will reduce the total system real power loss. IPFC consists of two series converters. One is named as Master and the other is slave. Optimal placement is applicable to Master line. Slave line can be any line which is closest and offers better electrical performance for the whole system.

The sensitivity factors $b_1{}^u$ is an indication of reduction in total system loss presented in Table 3 (method-1). For voltage magnitude control, line-5 is suitable as its sensitivity is the least. The magnitude of sensitivity factors of total system real power loss

Table 3
Sensitivities of 5-bus system using different methods.

L. No	OUPFC						
	Method-1				Method-2		
	b_1^o	b_2^o	b_3^o	b_4^o	c_1^o	c_2^o	c_4^o
1	5.8680	**10.7532**	0.0226	**11.6055**	**−0.6257**	−2.3363	−2.3058
2	4.3867	9.4317	0.0226	9.9992	−0.0357	−1.0725	−1.0444
3	5.1716	7.4567	0.0226	8.5937	0.9474	**4.0746**	**4.0133**
4	−0.6102	−1.1308	−0.0053	−1.2612	−0.4095	−2.4411	−2.3999
5	**−2.1869**	−5.7992	**−0.0215**	−6.2848	−0.2321	−1.2537	−1.2953
6	-0.8189	0.9053	0.0066	1.0939	0.5042	2.5421	2.6582

with respect to phase angle of UPFC (b_2^o) of OUPFC placed in line-1 is the highest followed by line-2 and with respect to phase angle of PST (b_4^o) of OUPFC placed in line-1 is the highest followed by line-2. This indicates that placement of OUPFC in line-1 will reduce the total system real power loss better compared to the placement in other lines which is a positive value. This indicates that placement of OUPFC in line-1 with negative phase shift will reduce the total system real power loss. The sensitivity factor b_3^o is almost same for each line, which is due to uniform voltage profile of the system. The sensitivity for line-5 is the highest negative, followed by line-4.

Method-1 does not consider the loading of the lines. Hence, it is not suitable for congestion management. In the event of congestion, for secure operation of the system, it is important to alleviate the overloads instead of reducing the losses in the system. This shows that method-1 is only appropriate for the placement of this device when there is no congestion.

performance index with respect to device parameters are obtained in Method 2. Objective here is to bring the power flow in line-3 to below 8.0 p.u, while the remaining lines also should not cross the limit.

The sensitivities of the real power flow PI with respect to UPFC control parameters are shown in Table 1 (method-2). The objective of placing UPFC is to divert the power from line-3 and to bring the power flow to within the limits. However, c_1^u for line-1 is the most negative and thus suitable for PI reduction with control of V_T. Table 1 (column 8, c_2^u) shows that placement of UPFC in line-3 is more sensitive than the placement in other lines. This sensitivity is positive which indicates that phase angle shift of the UPFC should be negative i.e., similar to placing an inductor in series with the line-3. Placing of UPFC in line-3 reduces the load of line-3 (heavily loaded) but increases the load of lines-4 and-5 which are under-loaded. Table 1 also shows that the placement of UPFC in line-6 with negative phase angle control is the next choice as the magnitude of sensitivity factors is the second highest. The sensitivity factor c_3^u is always zero because it cannot control the real power flow of the line since it is at 90° phase with input voltage. Optimal location for placement of UPFC is line 3 as sensitivity with respect to phase angle control is more effective than the voltage control.

The sensitivities of the real power flow PI with respect to IPFC control parameters are shown in Table 2 (method-2). It can be observed from Table 2 that the sensitivity c_1^u of PI with respect to voltage control for line-4 is the least. Column 7 (c_2^u) of Table 2 shows that placement of IPFC in line-3 is the most sensitive. This sensitivity is positive which indicates that phase angle shift of the IPFC should be negative. Placing of IPFC in line-3 will reduce the load of line-3 (heavily loaded) and placement of IPFC in line-6 (connecting bus-4 and 5) is the next choice as the magnitude of sensitivity factor is the second highest. As both 4th and 5th buses are generator buses, IPFC should not be placed in line-6. Placement of IPFC in line-1 (bus 1 to bus 2) since it is the third choice, also reduces the total system real power loss, and hence, used as slave line of IPFC. Therefore line-3 (from bus 1 to bus 4) has been chosen as the master line.

The sensitivities of the real power flow PI with respect to OUPFC control parameters are shown in Table 3 (method-2). Sensitivity factor c_1^o for line-1 is the least and thus suitable for PI reduction with control of magnitude of inserted voltage. Table 3 (columns 9 and 11, c_2^o, c_4^o) shows that placement of OUPFC in line-3 is more sensitive than other lines. This sensitivity is positive which indicates that phase angle shift of the OUPFC should be negative. Placement of OUPFC in line-3 reduces the load of line-3 (heavily loaded) but it increases the load of lines-4 and 5 which are under-loaded. Table 3 also shows that the placement of OUPFC in line-6 with negative phase angle control is the next choice as the magnitude of sensitivity factors is the second highest. Sensitivity

To check the effectiveness of the method-2, the line-loading has been changed by changing the generation schedule. At bus 4, generation capacity is decreased to 400 MW from 750 MW while keeping the load constant at buses 1 and 2. The change in generation is compensated by slack bus generator.

The sensitivity factors calculated for UPFC control parameters are given in Table 4 (sub columns 4 and 5). The magnitude of sensitivity of PI with respect to voltage control of UPFC for line-1 is the least but the value is less than that obtained in Table 1. The magnitude of sensitivity of PI with respect to phase angle of UPFC for line-3 is still higher than others lines but the value is less than that obtained in Table 1. The absolute value of sensitivity c_2^u corresponding to line-6 is the second highest. In such cases, decision to place UPFC is taken as per the sensitivity with respect to phase angle control c_2^u. The UPFC in this case should be placed in line-3.

The sensitivity factors computed for IPFC control parameters are given in Table 4 (columns 6 and 7). The magnitude of sensitivity of PI with respect to voltage control of UPFC for line-4 is the highest but the value is less than that obtained in Table 1. The magnitude of sensitivity of PI with respect to phase angle of IPFC for line-3 is the highest but the value is less than that obtained in Table 1. The absolute value of sensitivity c_2^I corresponding to line-6 is the second highest. The placement of IPFC in line-6 is the next choice with negative phase shift. In such cases, decision to place IPFC is taken as per the sensitivity with respect to phase angle control c_2^I. The master line of IPFC in this case should be placed in line-3. Slave line is Line-1 which connects bus 1 to bus 2.

The sensitivity factors calculated for OUPFC control parameters are given in Table 4 (columns 8, 9 and 10). The magnitude of sensitivity of PI with respect to voltage control of OUPFC for line-1 is the highest but the value is less than that obtained in Table 3. The magnitude of sensitivity factors of PI with respect to phase

Table 4
Performance index sensitivities for different loading limits.

Line-*k*		Power flows	UPFC		IPFC		OUPFC		
No	*i–j*	Base case	c_1^u	c_2^u	c_1^I	c_2^I	c_1^o	c_2^o	c_4^o
1	1–2	−0.149	**−0.398**	−1.338	−0.410	−1.339	**−0.398**	−1.338	−1.334
2	1–3	−6.767	0.002	−0.496	−0.500	−0.298	0.002	−0.496	−0.481
3	1–4	**−8.085**	0.712	**2.325**	0.135	**2.806**	0.712	**2.325**	**2.318**
4	2–5	−5.149	−0.204	−1.380	**−0.506**	−1.200	−0.204	−1.380	−1.357
5	3–4	3.024	−0.151	−0.641	−0.215	−0.654	−0.151	−0.641	−0.668
6	4–5	−1.298	0.296	1.468	0.332	1.475	0.296	1.468	1.548

angles of OUPFC for line-3 is still higher but the value is less than that obtained in Table 3. The absolute value of sensitivity factors c_2^o and c_4^o corresponding to line-6 each are the second highest. In such cases, decision to place OUPFC is taken as per the sensitivity factors with respect to phase angle controls c_2^o and c_4^o. The OUPFC in this case should be placed in line-3.

IEEE 14-bus test system

The IEEE 14-bus test system [44] consists of five generator buses, eleven load buses and twenty transmission lines shown in Fig. 15. The line flow limit is set to 120 MW. Bus 1 is the reference bus. Base MVA for the system is considered as 100MVA.

The generalized approach is tested on IEEE 14-bus system also. The sensitivities are calculated for each control parameters of UPFC, IPFC and OUPFC placed in every line in turns are shown in Tables 5–7.

Table 5 (method-1), presents the computed sensitivity factors power loss. The sensitivity of total system real power loss with respect to phase angle (b_2^u) when UPFC is placed in line-7 is the highest followed by line-2. This indicates that placement of UPFC in line-7 with positive phase shift reduces total real power loss. The sensitivity factor b_3^u is almost same for all lines, which is an indication of uniform voltage profile throughout the system.

Table 6 (method-1), presents the computed sensitivity factors b_1^I and b_2^I of the lines when IPFC is placed in each line in turn. Line-7 is suitable for voltage magnitude control b_1^I, as its sensitivity is the least. The absolute value of sensitivity with respect to phase angle (b_2^I) when IPFC placed in line-7 is the highest. This indicates that placement of IPFC in line-7 reduces the total system real power. This indicates that placement of IPFC in line-7 with positive phase shift reduces the total real power loss.

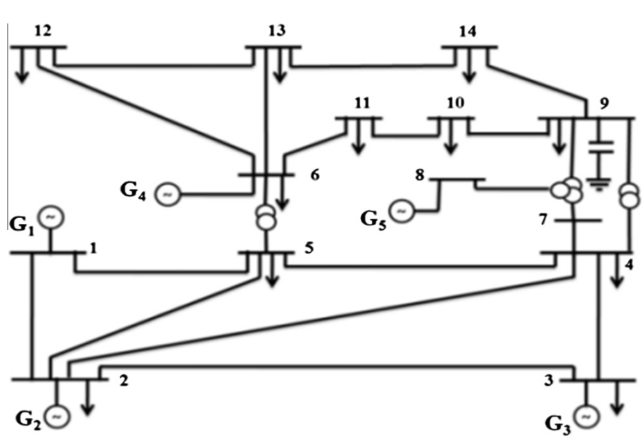


Fig. 15. IEEE 14-bus system.

Table 5
Sensitivities of 14-bus system using different methods.

Line-*k*		UPFC					
		Method-1			Method-2		
No	*i–j*	b_1^u	b_2^u	b_3^u	c_1^u	c_2^u	c_3^u
1	1–2	0.1764	−1.3100	−0.0162	**−0.0549**	**−0.1286**	0.0
2	1–5	−0.5338	**−1.3407**	**−0.0162**	0.0532	**0.1291**	0.0
3	2–3	−0.3183	0.0282	−0.0471	0.0056	0.0091	0.0
4	2–4	**−0.7204**	−0.1276	−0.0471	−0.0425	−0.0823	0.0
5	2–5	**−0.8659**	−1.3176	−0.0471	**−0.0493**	−0.0974	0.0
6	3–4	−0.3475	−0.0181	0.0175	0.0202	0.0360	0.0
7	4–5	−0.5251	**−4.9408**	0.0653	−0.0052	0.0045	0.0
8	4–7	0.3533	0.1631	0.0653	−0.0081	−0.0333	0.0
9	4–9	0.1808	0.1222	0.0653	0.0049	0.0177	0.0
10	5–6	0.2993	1.1238	−0.0144	0.0038	0.0145	0.0
11	6–11	−0.4013	0.1668	−0.0148	0.0016	0.0057	0.0
12	6–12	−0.2426	0.2465	−0.0148	−0.0017	0.0007	0.0
13	6–13	−0.1903	0.3461	−0.0148	−0.0126	−0.0061	0.0
14	7–8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0
20	13–14	0.0675	0.0150	0.0310	−0.0006	0.0013	0.0

Table 6
Sensitivities of 14-bus system using different methods.

Line-*k*		Power flows	IPFC			
			Method-1		Method-2	
No	*i–j*	Base case	b_1^I	b_2^I	c_1^I	c_2^I
1	1–2	**1.5690**	1.6374	−1.1475	−0.0549	−0.1286
2	1–5	0.7549	0.1692	**−1.3969**	0.0532	**0.1291**
3	2–3	0.7325	0.3572	−0.0864	0.0479	0.0019
4	2–4	0.5628	−0.1974	−0.1700	−0.0098	−0.0849
5	2–5	0.4136	−0.4837	−1.3729	−0.0254	−0.1009
6	3–4	−0.2327	−0.5870	−0.0104	0.0037	0.0366
7	4–5	−0.6247	**−1.1277**	**−4.9813**	**−0.0551**	0.0011
8	4–7	0.3002	0.6577	0.2981	0.0171	−0.0221
9	4–9	0.1679	0.3411	0.1512	0.0181	0.0201
10	5–6	0.4570	0.6223	1.1936	0.0306	0.0203
11	6–11	0.0644	−0.3501	0.1166	0.0059	0.0015
12	6–12	0.0767	−0.1771	0.2039	0.0038	−0.0029
13	6–13	0.1728	−0.0467	0.2354	−0.0006	−0.0154
14	7–8	−0.0000	0.0369	0.1516	0.0031	0.0127
15	7–9	0.2936	0.5013	0.2096	0.0123	−0.0435
16	9–10	0.0612	−0.3011	−0.1009	−0.0061	−0.0146
17	9–14	0.1000	0.1099	0.1404	−0.0021	−0.0124
18	10–11	−0.0289	−0.2588	0.2105	−0.0061	−0.0050
19	12–13	0.0150	0.1027	−0.1275	0.0021	0.0007
20	13–14	0.0507	0.1123	−0.0172	0.0031	−0.0014

The sensitivity factors b_1^o is an indication of reduction in total system loss presented in Table 7 (method-1). Line-5 is suitable for voltage magnitude control, since its sensitivity is the most negative. The magnitude of sensitivity factors of total system real power loss with respect to phase angle of UPFC ($b_2{}^o$) of OUPFC placed in line-7 is the highest followed by line-2. Sensitivity with respect to phase angle of PST ($b_4{}^o$) of OUPFC placed in line-7 is the highest followed by line-5. This indicates that placement of OUPFC in line-7 with positive phase shift reduces the total system real power loss. The sensitivity factor $b_3{}^o$ is almost same for all the lines. The sensitivity for line 17 is the highest negative, followed by line-2. According to method-1, the best location is line-7.

Method-1 does not consider the loading of the lines and hence, it is only appropriate for the placement of this device when there is no congestion.

From the load flow result in Table 6 (column 2, base case), the real power flow in line-1 is 1.5690 p.u, which is more than its line load limit of 1.2 p.u. Partial derivative of real power flow performance index with respect to device parameters are obtained in Method 2. Objective here is to bring the power flow in line-1 to below 1.2 p.u and to see that the remaining lines also should not cross their respective limits.

The sensitivities of the real power flow PI with respect to UPFC control parameters are shown in Table 5 (method-2). The objective of placing UPFC is to divert the power from line-1 and to bring the power flow within the limits. However, $c_1{}^u$ for line-1 is the least and thus suitable for PI reduction with control of V_T. Column 8 ($c_2{}^u$) shows that placement of UPFC in line-2 is the most sensitive. This sensitivity is positive which indicates that phase angle shift of the UPFC should be negative (similar to placing a capacitor in series with the line 2). Table 5 also shows that the placement of UPFC in line-1 with positive phase angle control is the next choice as its magnitude of sensitivity factor is the second highest. The sensitivity factor $c_3{}^u$ is always zero. Optimal location for Placement of UPFC

control parameters are shown in Table 6 (method 2). It can be observed from Table 6 that the sensitivity $c_1{}^u$ of PI with respect to voltage control for line-7 is the least value. Column 7 ($c_2{}^u$) of Table 6 shows that placement of IPFC in line-2 is most sensitive. Hence, place a capacitive branch of IPFC. Placing IPFC in line-2 reduces the load of line-1 (heavily loaded) and placement of IPFC in line-1 (bus 1 to bus 2) is the next choice as the magnitude of sensitivity factor is the second highest. Hence, line-1 is used as slave line for IPFC. Therefore line-2 (bus 1 to bus 5) has been chosen as the master line.

The sensitivities of the real power flow PI with respect to OUPFC control parameters are shown in Table 7 (method-2). The sensitivity factor $c_1{}^o$ for line-1 is the most negative and thus suitable for PI reduction with control of magnitude of inserted voltage. Columns 9 and 11, ($c_2{}^o$, $c_4{}^o$) show that placement of OUPFC in line-2 is more sensitive than the placement in any other line. This sensitivity is positive which indicates that phase angle shift of the OUPFC should be negative. Placement of OUPFC in line-2 will reduce the loading of line-1 (heavily loaded) but increases the loading of lines-3 and 4 which are under-loaded. Table 7 also shows that the placement of OUPFC in line-1 with positive phase angle control is the next choice as the magnitude of sensitivity factors is the second highest. The sensitivity factor $c_3{}^o$ is always zero.

Comparison of results

The results obtained using proposed method for 5-bus system to minimize total active power loss when UPFC and OUPFC located between bus 1–2 and IPFC located between bus 1–4 are presented in Table 8. Also, for minimization of Performance Index, all the

Table 8
Comparison of test results for 5 bus system.

S. no	Device	Ploss (MW)		PI	
		Proposed	PSO [31]	Proposed	PSO [31]
1	UPFC	1.001	1.008	0.276	0.286
2	IPFC	1.000	1.002	0.288	0.291

Table 9
Comparison of results for 14 bus system.

S. no	Device	Ploss (MW)		PI	
		Proposed	GA [32]	Proposed	GA [32]
1	UPFC	1.125	1.128	0.304	0.334
2	IPFC	1.116	1.120	0.311	0.361
3	OUPFC	1.009	1.020	0.327	0.360

Table 7
Sensitivities of 14-bus system.

L. No	OUPFC						
	Method-1				Method-2		
	$b_1{}^o$	$b_2{}^o$	$b_3{}^o$	$b_4{}^o$	$c_1{}^o$	$c_2{}^o$	$c_4{}^o$
1	0.1764	−1.3100	−0.0162	−1.5366	**−0.0549**	**−0.1286**	**−0.1363**
2	−0.5338	**−1.3407**	**−0.0162**	−1.4449	0.0532	**0.1291**	**0.1369**
3	−0.3183	0.0282	−0.0471	−0.0783	0.0056	0.0091	0.0099
4	−0.7204	−0.1276	−0.0471	−0.3388	−0.0425	−0.0823	−0.0895
5	**−0.8659**	−1.3176	−0.0471	**−1.5990**	−0.0493	−0.0974	−0.1059
6	−0.3475	−0.0181	0.0175	−0.0209	0.0202	0.0360	0.0400
7	−0.5251	**−4.9408**	0.0653	**−4.0977**	−0.0052	0.0045	0.0035
8	0.3533	0.1631	0.0653	0.2093	−0.0081	−0.0333	−0.0349
9	0.1808	0.1222	0.0653	0.1452	0.0049	0.0177	0.0186
10	0.2993	1.1238	−0.0144	1.1864	0.0038	0.0145	0.0152
11	−0.4013	0.1668	−0.0148	0.0017	0.0016	0.0057	0.0064
12	−0.2426	0.2465	−0.0148	0.1413	−0.0017	0.0007	0.0002
13	−0.1903	0.3461	−0.0148	0.2101	−0.0126	−0.0061	−0.0098
14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15	0.2452	0.3261	0.0000	0.3982	−0.0092	−0.0337	−0.0371
16	−0.3485	−0.0462	−0.0165	−0.2800	−0.0102	−0.0098	−0.0128
17	0.0242	0.1998	**−0.0165**	0.1622	−0.0096	−0.0072	−0.0100
18	−0.2363	0.1866	0.0039	0.1380	−0.0042	−0.0072	−0.0084
19	0.0903	−0.1158	0.0774	0.3133	0.0011	0.0016	0.0020
20	0.0675	0.0150	0.0310	0.1069	−0.0006	0.0013	0.0011

three devices are optimally located between bus 1–4 and PI values are presented in Table 8. The results of this test system are compared with the existing results of PSO algorithm [31].

Similarly for 14-bus system, the results of total active power loss and PI when all the three devices located between bus 4–5 and bus 1–5 respectively are presented in Table 9. The results of this test system are compared with the existing results of Genetic Algorithm [32].

For both the test systems, results of proposed method is better than the results of exists in the literature.

From the results it was observed that sensitivity based generalized approach has given best location for the FACT devices placement, which resulted in maximum reduction in the line overload performance index values.

Conclusions

A generalized method has been developed to determine suitable location for placement of any FACT device. According to the proposed method, objective function is partially differentiated with respect to control parameters of the FACT devices. This is implemented for two objective functions and validated for three devices, UPFC, IPFC and OUPFC. The objective functions considered here are reduction of total system real power loss and reduction of real power flow performance index.

Partial derivatives of total system real power loss with respect to system parameters are obtained in Method-1, which is sufficient to determine the location as long as there is no congestion. In congested system, the suitable location of FACTs device can be decided based on the partial derivative of real power flow performance

magnitude of the voltage.

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