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Aircraft Control System Using LQG and LQR Controller with Optimal Estimation-Kalman Filter Design

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Abstract

This paper, describes a LQG and LQR robust controller for the lateral and longitudinal flight dynamics of an aircraft control system. The controller is used in order to achieve robust stability and good dynamic performance against the variation of aircraft parameters. The application of the proposed LQG and LQR robust control scheme is implemented through the simulation. The proposed robust controller for aircraft stability is designed using Matlab/Simulink program. Simulation results confirm the performance of the proposed controller for aircraft control system. Since the time of its introduction, the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. For example, to determine the velocity of an aircraft or sideslip angle, one could use a Doppler radar, the velocity indications of an inertial navigation system, or the relative wind information in the air data system. Rather than ignore any of these outputs, a Kalman filter could be built to combine all of this data and knowledge of the various systems dynamics to generate an overall best estimate of pitch, roll and sideslip angle.

Selection and peer-review under responsibility of Airworthiness Technologies Research Center, Beihang University/NLAA.

Keywords: Aircraft motion; LQG control; LQR control; lateral stability; longitudinal stability; State estimator Kalman filter.

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Nomenclature

γ	Path angle
θ	Pitch angle
α	Angle of attack
ϕ	Roll angle
β	Sideslip angle
q	Pitch rate
U_0	Longitudinal velocity
m	Aircraft mass
δ_e	Elevator deflection
δ_r	Rudder deflection
δ_a	Aileron deflection

1. Introduction

The Feedback control systems are widely used in manufacturing, mining, automobile and military hardware applications. In response to demands for increased efficiency and reliability, these control systems are being required to deliver more accurate and better overall performance in the face of difficult and changing operating conditions. In order to design control systems to meet the demands of improved performance and robustness when controlling complicated processes, control engineers will require new design tools and better underlying theory. In particular, a standard method of improving the performance of a control system is to add extra sensors and actuators. This necessarily leads to a multi-input multi-output control system. Thus, it is a requirement for any modern feedback control system design methodology that it be able to handle the case of multiple actuators and sensors. Linear Quadratic Gaussian optimal control theory (LQG) is one of the major achievements of the modern control area. This controller design methodology enables a controller to be synthesized which is optimal with respect to a specified quadratic performance index. Furthermore, this theory takes into account the presence of Gaussian white noise disturbances acting on the system. Indeed, in many practical control problems, it is straightforward to translate the required performance objective into a problem of minimizing a quadratic cost functional. Also, in many practical control problems, the system is subject to disturbances and measurement noise which are most naturally modeled as stochastic white noise processes.

The LQG controller design methodology based on the Kalman filter who in 1960 published his famous paper describing a recursive solution to the discrete-data linear filtering problem. A more complete introductory discussion can be found in [1] which also contains some interesting historical narrative. More extensive references include [2], [3] and [4]. It has also been used for motion prediction [7] and it is used for multi-sensor . In practice, although it is possible to obtain process models either from first principles or from experimental measurements, these models will always be subject to errors. Thus, the control system needs to be designed to be robust against these modeling errors.

1.1 Aircraft control and movement

There are three primary ways for an aircraft to change its orientation relative to the passing air. *Pitch* (movement of the nose up or down), *Roll* (rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) and *Yaw* (movement of the nose to left or right.) Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero. [5]

2. Aircraft longitudinal dynamics

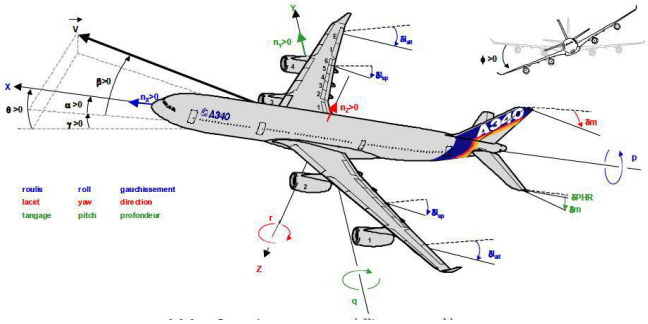

Fig.1. Aerodynamic reference

1.2. Equations of movements:

The general equations of the movement are governed by the equations of mechanics

$$\begin{cases} m\dfrac{\overrightarrow{du}}{dt} = \sum \overrightarrow{F_e} \\ \dfrac{\overrightarrow{dC}}{dt} = \sum \overrightarrow{M_e} \end{cases} \tag{1}$$

1.2.1. Equation of longitudinal motion:

$$\beta = p = r = \Phi = 0 \tag{2}$$

Longitudinal equations can be rewritten as:

$$\begin{cases} \dot{u} = \frac{X_u}{m}u + \frac{X_w}{m}w - \frac{g\cos\Theta_0}{m}\theta + \Delta X^c \\ \dot{w} = \frac{Z_u}{m-Z_{\dot{w}}}u + \frac{Z_w}{m-Z_{\dot{w}}}w + \frac{Z_q+mU_0}{m-Z_{\dot{w}}}q - \frac{mg\sin\Theta_0}{m-Z_{\dot{w}}}\theta + \Delta Z^c \\ \dot{q} = \frac{[M_u+Z_u\Gamma]}{I_{yy}}u + \frac{[M_u+Z_u\Gamma]}{I_{yy}}w + \frac{[M_q+(Z_q+mU_0)\Gamma]}{I_{yy}} \\ \quad -\frac{mg\sin\Theta_0\,\Gamma}{I_{yy}}\theta + \Delta M^c \\ \dot{\theta} = q \end{cases} \tag{3}$$

With:

$$\Delta X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\delta_p$$
$$\Delta Z^c = \frac{Z_{\delta_e}}{m - Z_{\dot{w}}}\delta_e + \frac{Z_{\delta_P}}{m - Z_{\dot{w}}}\delta_p$$
$$\Delta M^c = \frac{M_{\delta_e} + Z_{\delta_e}\Gamma}{I_{yy}}\delta_e + \frac{M_{\delta_p} + Z_{\delta_p}\Gamma}{I_{yy}}\delta_p \tag{4}$$

Rewrite in state space form as:

-Since $u \approx 0$ in this mode, then $\dot{u} \approx 0$ and can eliminate the X force equation:

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{Z_w}{m - Z_{\dot{w}}} & \frac{Z_q + mU_0}{m - Z_{\dot{w}}} & \frac{-mg\sin\Theta_0}{m - Z_{\dot{w}}}\\ \frac{[M_w + Z_w\Gamma]}{I_{yy}} & \frac{[M_q + (Z_q + mU_0)\Gamma]}{I_{yy}} & \frac{-mg\sin\Theta_0}{I_{yy}}\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\Delta Z^c\\ \Delta M^c\\ 0\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{Z_w}{m} & U_0 & -g\sin\Theta_0\\ \frac{\left[M_w + Z_w\frac{M_{\dot{w}}}{m}\right]}{I_{yy}} & \frac{\left[M_q + (mU_0)\frac{M_{\dot{w}}}{m}\right]}{I_{yy}} & \frac{-mg\sin\Theta_0}{I_{yy}}\frac{M_{\dot{w}}}{m}\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\Delta Z^c\\ \Delta M^c\\ 0\end{bmatrix} \tag{6}$$

The transfer function can be represented in state-space form and output equation as state by equation

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}-0.3149 & 235.8928 & 0\\ -0.0034 & -0.4282 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}-5.5079\\ 0.0021\\ 0\end{bmatrix}\delta_e \tag{7}$$

$$y = [\,0\ \ 0\ \ 1\,]\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + [0] \tag{8}$$

This work presents investigation into the development of pitch control schemes for pitch angle and pitch rate of an aircraft systems. Pitch control systems with full state feedback controller are investigated. A modern controller (LQG) control the pitch of an aircraft system. Performance of one control strategy with respect to the pitch. Simulation results are shown in Fig. 2

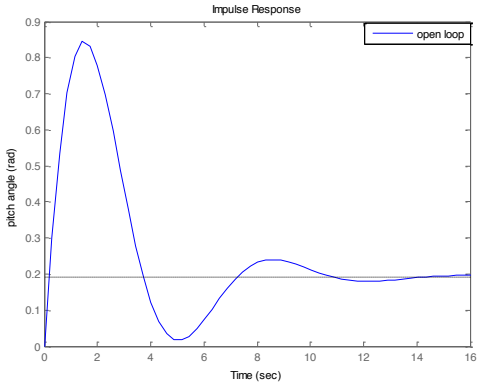

Fig.2. Open loop Impulse Response (Pitch angle)

$X = [u, \omega, q, \gamma]^T$ and $\gamma = \theta - \alpha$ represent flight path angle, with $\alpha = \omega$, $u = \begin{bmatrix} \delta_e \\ \delta_p \end{bmatrix}$

The input (elevator deflection angle, δ_e) will be 0.2 rad (11 degrees), and the output is the pitch angle (theta).

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The input (elevator deflection angle, δ_e) will be 0.2 rad (11 degrees), and the output is the pitch angle (theta).

There are three types of possible lateral-directional dynamic motion: roll subsidence mode, Dutch roll mode, and spiral mode.

3. Aircraft lateral dynamics

Using a procedure similar to the longitudinal mode, we can develop the equation of motion for the lateral dynamics.

$$\dot{x} = Ax + Bu, \qquad x = \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{9}$$

$x^T = [\beta\ p\ r\ \phi]^T$: state vector
$u^T = [\delta_a\ \delta_r]^T$: control vector
δ_a, δ_r: aileron and rudder deflection
β, ϕ: sideslip and roll angle
p, r: roll and yaw rate

$$A = \begin{bmatrix} \frac{Y_\beta}{U_0} & \frac{Y_p}{U_0} & \frac{Y_r}{U_0} & \frac{g\cos\theta_0}{U_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{Y_{\delta_r}}{U_0} & \frac{Y_{\delta_a}}{U_0} \\ L_{\delta_r} & L_{\delta_a} \\ N_{\delta_r} & N_{\delta_a} \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

If we assume that the measurable outputs are the sideslip angle β and roll angle ϕ , the matrixes A, B and C are:

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.5980 & -0.1150 & -0.0318 & 0 \\ -3.0500 & 0.3880 & -0.4650 & 0 \\ 0 & 0.0805 & 1.000 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.0729 & 0.000 \\ -4.7500 & 0.00775 \\ 0.15300 & 0.1430 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

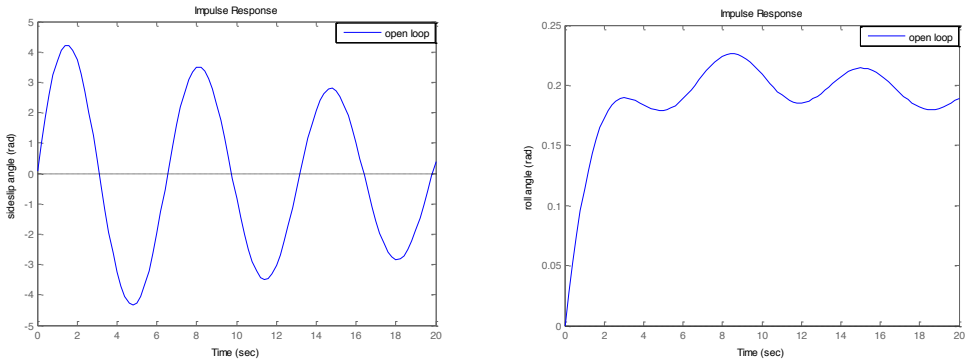


Fig.3. (a) Open loop Impulse Response (Sideslip angle); (b) Open loop Impulse Response (Roll angle)

4. Linear Quadratic Gaussian Controller

Linear Quadratic Gaussian (LQG) control is a modern state space technique for designing optimal dynamic regulators. It enables you to trade off regulation performance and control effort, and to take into account process and measurement noise. Like pole placement, LQG design requires a state-space model of the plant. This section focuses on the discrete-time case. To form the LQG regulator, simply connect the Kalman filter and LQ-optimal gain K as shown below:

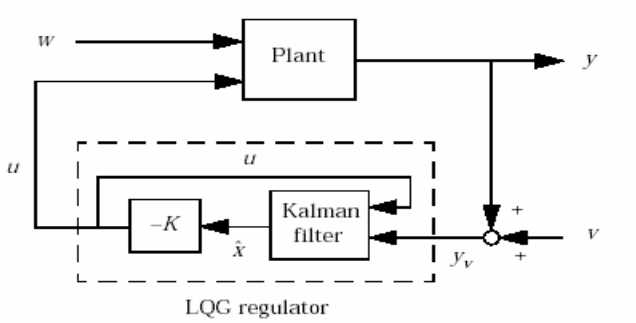


Fig.4. Block diagram of LQG Controller

This regulator has state-space equations

$$\frac{d}{dt}\hat{x} = [A - LC - (B - LD)K]\hat{x} + Ly_v \qquad (12)$$
$$u = -Kx$$

The goal is to regulate the output y around zero. The plant is subject to disturbances and is driven by controls. The regulator relies on the noisy measurements $y_v = y + v$ to generate these controls. The plant state and measurement equations are of the form

$$\dot{x} = Ax + Bu + Gw$$
$$y_v = Cx + Du + Hw + v \qquad (13)$$

and both w and v are modeled as white noise.
The LQG regulator consists of an optimal state-feedback gain and a Kalman state estimator. You can design these two components independently as shown next.

4.1. Optimal State-Feedback Gain

In LQG control, the regulation performance is measured by a quadratic performance criterion of the form

$$J(u) = \int_0^\infty \{x^T Qx + 2x^T Nx + u^T Ru\} \qquad (14)$$

The weighting matrices Q, N and R are user specified and define the trade-off between regulation performance (how fast goes to zero) and control effort. The first design step seeks a state feedback law that minimizes the cost function. This gain is called the *LQ-optimal* gain.

4.2. Kalman State Estimator

As for pole placement, the LQ-optimal state feedback $u = -kx$ is not implementable without full state measurement. However, we can derive a state estimate $\hat{x}$ such that $u = -k\hat{x}$ remains optimal for the output- feedback problem. This state estimate is generated by the Kalman filter.

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du) \tag{15}$$

With inputs u (controls) and y_v (measurements).The noise covariance data

$$E(ww^T) = Q_n\ , \qquad E(vv^T) = R_n\ , \qquad E(wv^T) = N_n \tag{16}$$

Determines the Kalman gain L through an algebraic Riccati equation.
The Kalman filter is an optimal estimator when dealing with Gaussian white noise. Specifically, it minimizes the asymptotic covariance of the estimation error $x - \hat{x}$.

$$\lim_{t\to\infty} E((x-\hat{x})(x-\hat{x})^T) \tag{17}$$

The goal is to regulate the plant output y around zero. The input disturbance d is low frequency with power spectral density (PSD) concentrated below 10 rad/sec. For LQG design purposes, it is modeled as white noise driving a low-pass filter with a cutoff at 10 rad/sec, as this picture shows.(fig 05-06)
There is some measurement noise n , with noise intensity given by

$$E(n^2) = 0.01 \tag{18}$$

Use the cost function

$$J(u) = \int_0^\infty (10y^2 + u^2)dt \tag{19}$$

to specify the trade-off between regulation performance and cost of control. Note that an open-loop state-space model is:

$$\begin{aligned} \dot{x} &= Ax + Bu + Bd \quad \text{(state equations)} \\ y_v &= Cx + n \quad \text{(measurements)} \end{aligned} \tag{20}$$

Simulation results are shown in Fig. 5-6

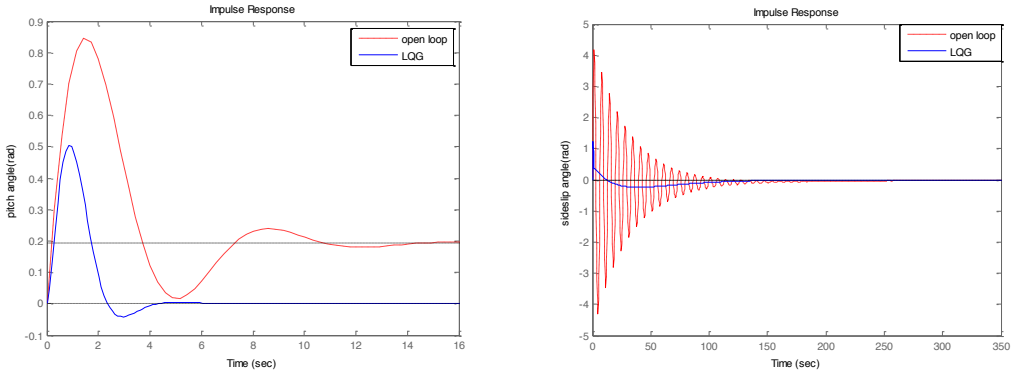


Fig.5. (a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Pitch angle), (b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Sideslip angle)

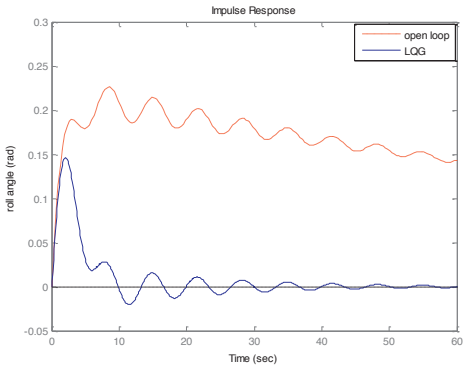


Fig.6. Comparison of Open- and Closed-Loop Impulse Response for the LQG Example (Roll angle)

5. Linear Quadratic Regulator Controller

Modern control theory has made a significant impact on the aircraft industry in recent years [10]. LQR is a method in modern control theory that used state-space approach to analyze such a system. Using state space methods it is relatively simple to work with a multi-output system. The system can be stabilized using full-state feedback system. The configuration of this control system is shown in Figure 08-09.

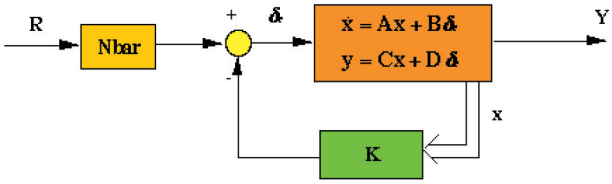


Fig.7. Full-state feedback controller with reference input

In designing LQR controller, *lqr* function in Matlab can be used to determine the value of the vector K which determined the feedback control law. This is done by choosing two parameter values, input $R = 1$ and $Q = C^T * C$ where C^T is the matrix transpose of C from state equation (6) and (11). The controller can be tuned by changing the nonzero elements in q matrix which is done in m-file code as obtained.

$$R = 1;$$
$$Q = [0\ 0\ 0; 0\ 0\ 0; 0\ 0\ x]; \tag{21}$$
$$K = lqr\ [A, B, Q, R];$$

Consequently, by tuning the value of $x = 500$, the following values of matrix K are obtained. If x is increased even higher, improvement to the response should be obtained even more. But for this case, the values of $x = 500$ is chosen because it satisfied the design requirements while keep x as small as possible.

In order to reduce steady state error of the system output, a value of constant gain *Nbar* should be added after the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, multiply that by the chosen gain K, and used a new value as the reference for computing the input. *Nbar* can be found using the user-defined function which can be used

in m-file code. The method used in simulation work is done by exported both value of matrix *K* and constant gain. For this controller design, the value of constant gain, *Nbar* are found to be, *Nbar = 100.*

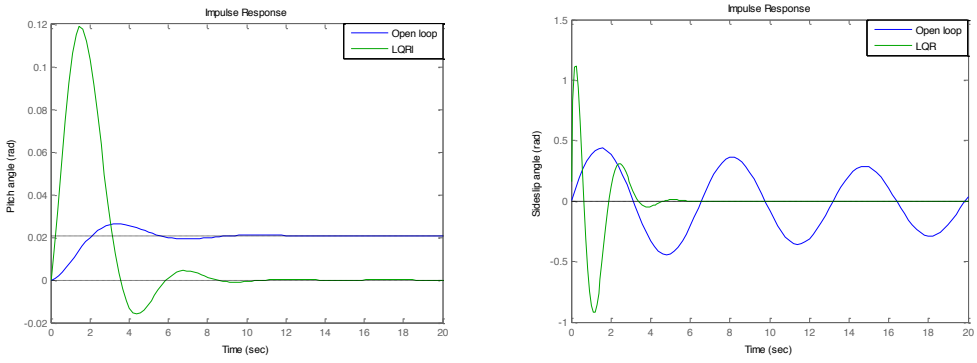


Fig.8. (a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Pitch angle), (b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Sideslip angle)

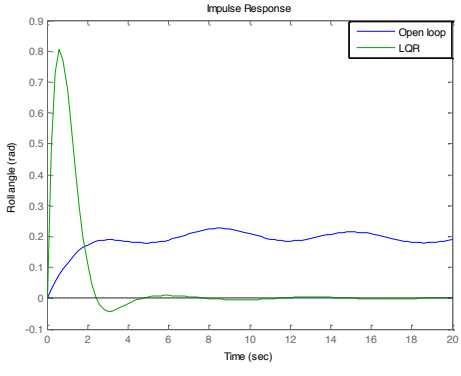


Fig.9. Comparison of Open- and Closed-Loop Impulse Response for the LQR Example (roll angle)

6. Kalman Filtering

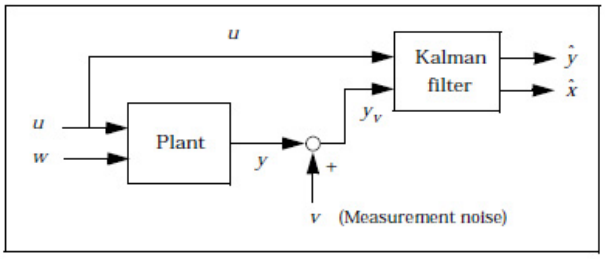
Consider the discrete plant

$$x(n+1) = Ax(n) + B\big(u(n) + w(n)\big)$$
$$y(n) = Cx(n) \tag{22}$$

with additive Gaussian noise $w(n)$on the input $u(n)$ and data.
Our goal is to design a Kalman filter that estimates the output $y(n)$ given the inputs $u(n)$ and the noisy output measurements

$$y_v(n) = Cx(n) + v(n) \tag{23}$$

where $v(n)$ is some Gaussian white noise.



Kalman estimator

Fig.10. Kalman estimator

6.1. Discrete Kalman Filter

The equations of the steady-state Kalman filter for this problem are given as follows.
Measurement update

$$\hat{x}(n/n) = \hat{x}(n/n-1) + M(y_v(n) - C\hat{x}\,(n/n-1)) \tag{24}$$

Time update

$$\hat{x}(n+1/n) = A\hat{x}(n) + Bu(n) \tag{25}$$

In these equations:

- $\hat{x}(n/n-1)$ is the estimate of $x(n)$ given past measurements up to $y_v(n-1)$
- $\hat{x}(n/n)$ is the updated estimate based on the last measurement $y_v(n)$

Given the current estimate $\hat{x}(n/n)$, the time update predicts the state value at the next sample $n+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_v(n-1)$. The correction term is a function of the *innovation*, that is, the discrepancy.

$$y_v(n-1) - C\hat{x}(n/n-1) = C(x(n+1) - \hat{x}(n+1/n)) \tag{26}$$

between the measured and predicted values of $y(n+1)$. The innovation gain M is chosen to minimize the steady-state covariance of the estimation error given the noise covariances.

$$E(w(n)w(n)^T) = Q,\ \ E(v(n)v(n)^T) = R, \tag{27}$$

You can combine the time and measurement update equations into one state-space model (the Kalman filter).

$$\hat{x}(n+1/n) = A(I - MC)\hat{x}(n/n-1) + [B\ AM]\begin{bmatrix} u(n) \\ y_v(n) \end{bmatrix}$$
$$\hat{y}(n/n) = C(I - MC)\hat{x}(n/n-1) + CMy_v(n) \tag{28}$$

This filter generates an optimal estimate $\hat{y}(n/n)$ of $y(n)$.
That the filter state is $\hat{x}(n/n-1)$
Simulation results are shown in Fig. 11-12

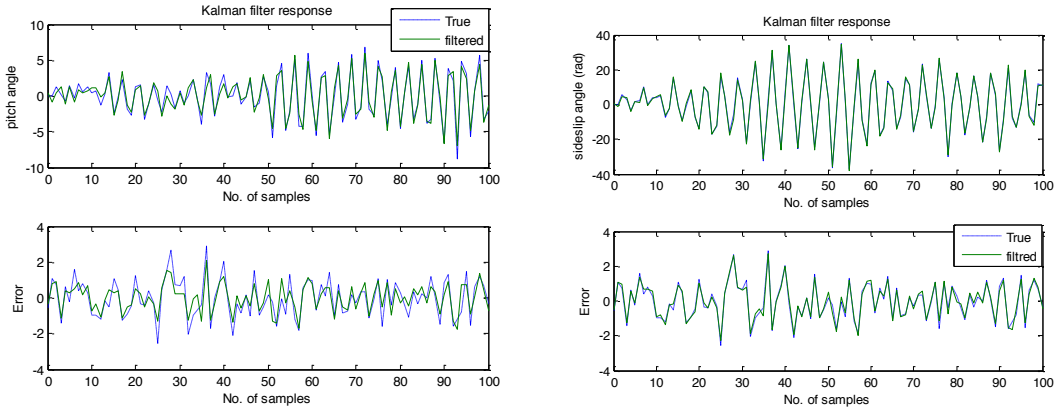


Fig.11. (a) Kalman Filter Response for the pitch angle θ, (b) Kalman Filter Response for the sideslip angle β

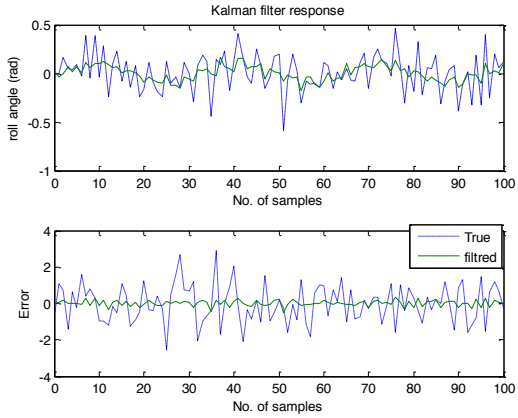


Fig.12. Kalman Filter Response for the roll angle ϕ

The first plot shows the true response y (dashed line) for the pitch angle θ and the filtered output y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid). This plot shows that the noise level has been significantly reduced. This is confirmed by the following error covariance computations.

7. Conclusion

The validated model of pitch, roll and sideslip control of an aircraft is very helpful in developing the control strategy for actual system. Pitch, roll and sideslip control of an aircraft is a system which requires a pitch, roll and sideslip controller to maintain the angle at it desired value. This can be achieved by reducing the error signal which is the difference between the output angle the desired angle. The control approach of LQR is capable on controlling the pitch angle, roll angle and sideslip angle of the aircraft

system for value of 0.2 radian (11.5 degree). Simulation and analysis results show that, LQR controller relatively give the better performance. For advanced work, effort can be devoted in developing more robustness control techniques, following by implement the proposed control algorithm to real plant for validating of the theoretical result.

Finally, the LQG gives a very good following to the outputs of plant with a steady shift error limited and the Kalman filter is an optimal estimator when dealing with Gaussian white noise. Optimal estimation provides an alternative rationale for the choice of observer gains in the current estimator which is based on observer performance in the presence of process noise and measurement errors.

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: *time update* equations and *measurement update* equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

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