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Kalman Filter based Target Tracking for Track While Scan Data Processing

K. David Solomon Raj
Lecturer, Dept.of Avionics
JNTUK-Kakinada, Andhra Pradesh
k.rajusolomon@gmail.com

I. Mohan Krishna
PG Scholar, Dept.of Avionics
JNTUK-Kakinada, Andhra Pradesh
mohankrish236@gmail.com

***Abstract*—The targets parameter to be measured for tracking are its relative position in range, azimuth angle, elevation angle and velocity. These parameters can be measured by tracking radar systems. Upon keeping the tracking of these measured parameters the tracker predict their future values. Fire control and missile guidance can be assisted through target tracking only. In fact missile guidance cannot be achieved without tracking the target properly. To predict target parameters (future samples) between scans, track while scan radar system sample each target once per scan interval by using sophisticated smoothing and prediction filters among which alpha-beta-gamma ($\alpha\beta\gamma$) and Kalman filters are commonly used. The principle of recursive tracking and prediction filters are proposed in this paper for two maneuvering targets (lazy and aggressive maneuvering), by implementing the second and third order one dimensional fixed gain polynomial filter trackers. Finally the equations for an n-dimensional multi state kalman filter are implemented and analyzed. In order to evaluate the performance of tracking filters the target considered in this paper is a Novator K100 Indian/Russian air-to-air missile designed to fly at Mach 4. In this paper the main objective of developing these filter tracking algorithmsis to reduce the measurement noise and tracking filter must be capable of tracking maneuvering targets with small residual (tracking errors).**

Keywords—alpha-beta-gamma ($\alpha\beta\gamma$), Kalman filters and Residual error

I. INTRODUCTION

Tracking radar systems are used to measure the target's relative position in range, azimuth angle, elevation angle, and velocity. By using the measured values of these parameters the target's future parameters can be predicted. Target tracking is the most important aspect of both military and civilian radar systems. In military radars, tracking is done for fire control and missile guidance; missile guidance is almost impossible without target tracking. Civilian airport traffic control radars, utilize tracking as a means of controlling incoming and departing airplanes.

Based on tracking parameter, target tracking is classified as range/velocity tracking and angle tracking. Based on nature of tracking, tracking radars are classified as continuous single-target tracking radars and multi-target track-while-scan (TWS) radars. In radar the goal is to estimate the range of objects (airplanes, ships, etc.) by analyzing the two-way transit timing of received echoes of transmitted pulses. Since the reflected pulses are unavoidably embedded in electrical noise, their measured values are randomly distributed, so that the transit time must be estimated. The measurements which contain information regarding the parameters of interest are often associated with a noisy signal. Without randomness, or noise, the problem would be deterministic and estimation would not be needed. [1]

This paper focuses on techniques and algorithms necessary for target tracking in two scenarios. The first case is to track a lazy maneuvering target and the second case is to track aggressive maneuvering target. The organization of the paper is as follows: Section 2 deals with the problem of target tracking. Section 3 deals with the actual scenario to perform prediction and tracking. Section 4 and 5 deals with the application ofalpha-beta, alpha-beta-gamma and Discrete-time Kalman filters for the linear state estimation problem for tracking a target.

II. PROBLEM OF STATE ESTIMATION

The filtering problem can be expressed as estimating the state X of a dynamicsystem.The state vector X usually describesthe state of target i.e. target position, velocity, and sometimes acceleration as state variables. The estimation of next state of the dynamical systems having uncertainties in its state and measurement can be done using filters. [2]

The real state of a system is never known and hidden by the environment. All information about the state is gained through state-dependent measurements of the environment.Both, measurements on the environment and evolution of the state over time, are distorted by noise (created by physical uncertainties and assumptions). They are corrupted by System Noise (state) and measurement noise. [3]

III. THE ACTUAL SCNERAIO

In order to perform the tracking the target which we are considering is a K100 missile which is carried by Unmanned Aerial Vehicle (UAV) flying at 1 Km/s.TheNovator K-100 is an Indian/Russian air-to-air missile designed as an "AWACS killer" at ranges up to 300–400 km (160-210 mi)[7]. Now the assumptions and considerations are as follows: It is assumed that the missile and UAV are moving in same direction and the initial estimate of the state of the missile is $X_0(\hat{X}_{k-1}) = [2, 1, 0]$which includes range, velocity and acceleration of the missile. Also the simulation result has been done for the two measurement models. The first model is assumed that the missile is flying with constant velocity given by the model

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equation $S = 0.32t$ and the second model is assumed that the missile is flying with acceleration $a = 0.32\ Km/s^2$ given by the model equation $S = 0.32t + 0.16t^2$. Also the simulation results displayed in section 4 and 5 shows predicted position and residual error for two cases: Lazy maneuvering and aggressive maneuvering. The noise considered here is white Gaussian noise with zero mean and variance of $\sigma^2 = 0.01$.

IV. FIXED GAIN TRACKING FILTERS

Fixed gain filters were the first rustic filters and the most famous implementation of fixed gain filters; suitable for noisy kinematic models are alpha-beta filter and alpha-beta-gamma filter. These two filters are special cases of one dimensional Kalman filter. Alpha-beta filter is one dimensional second order filter and alpha-beta-gamma filter is one dimensional third order filter. [1]

For the purpose of the discussion presented in this paper, the following notation is adopted and the fixed-gain filter equation is given by

$$\hat{X}_k(+) = \phi\hat{X}_{k-1}(+) + K[Z_k - G\phi\,\hat{X}_{k-1}(+)] \qquad (1)$$

Since the transition matrix assists in predicting the next state,

$$\hat{X}_k(-) = \phi\hat{X}_{k-1}(+) \qquad (2)$$

Substituting Eq. (2) into Eq. (1) yields

$$\hat{X}_k(+) = \hat{X}_{k-1}(+) + \underline{K}[Z_k - \underline{G}\,\hat{X}_k(-)] \qquad (3)$$

The term enclosed within the brackets on the right hand side of Eq. (3) is often called the residual (error) which is the difference between the measured input and predicted output. Eq. (3) means that the estimate of $\hat{X}_k(+)$is the sum of the prediction and the weighted residual.The term $\underline{G}\,\hat{X}_k(-)$ represents the prediction state. In the case of the α-β-γ estimator, $\underline{G}$ *is* row vector and is given by

$$\underline{G} = [1 \quad 0 \quad 0] \qquad (4)$$

and the gain matrix $\underline{K}$ is given by

$$\underline{K} = \begin{bmatrix} \alpha \\ \beta/T \\ \gamma/T^2 \end{bmatrix} \qquad (5)$$

For a third order filter the state transition matrix for a discrete linear time invariant system is ϕ is given by

$$\phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

The matrix given is often called the Newtonian matrix.

For any fixed gain filter Eq. (2) represents the prediction equation (time update) and Eq. (3) represents smoothed equation(measurement update).These are recursive filters in every iteration the prediction and smoothed equations are updated by substituting all the above mentioned values in Eq.(2) and Eq.(3).Here in fixed gain filters the gain matrix $\underline{K}$ is constant and is initially calculated and will be used in all iterations.

The α-β-γ tracker produces, on the *nth* observation, smoothed estimates for position, velocity, acceleration and a predicted position and velocity for the *(n+1)th*observation.The α-β tracker can follow an input ramp (constant velocity) with no steady state errors. However, a steady state error will accumulate when constant acceleration is present in the input.

An important sub-class of the α-β tracker is the critically damped filter, often called the fading memory filter. In this case, the filter coefficients are chosen on the basis of a smoothing factor ξ, where $0 \le \xi \le 1$.The gain coefficientsare given by

$$\alpha = (1-\xi); \qquad (7)$$

$$\beta = 1.5(1-\xi)^2(1+\xi); \qquad (8)$$

$$\gamma = (1-\xi)^3 \qquad (9)$$

The α-β-γ tracker filter steps are briefly given in Figure 1.

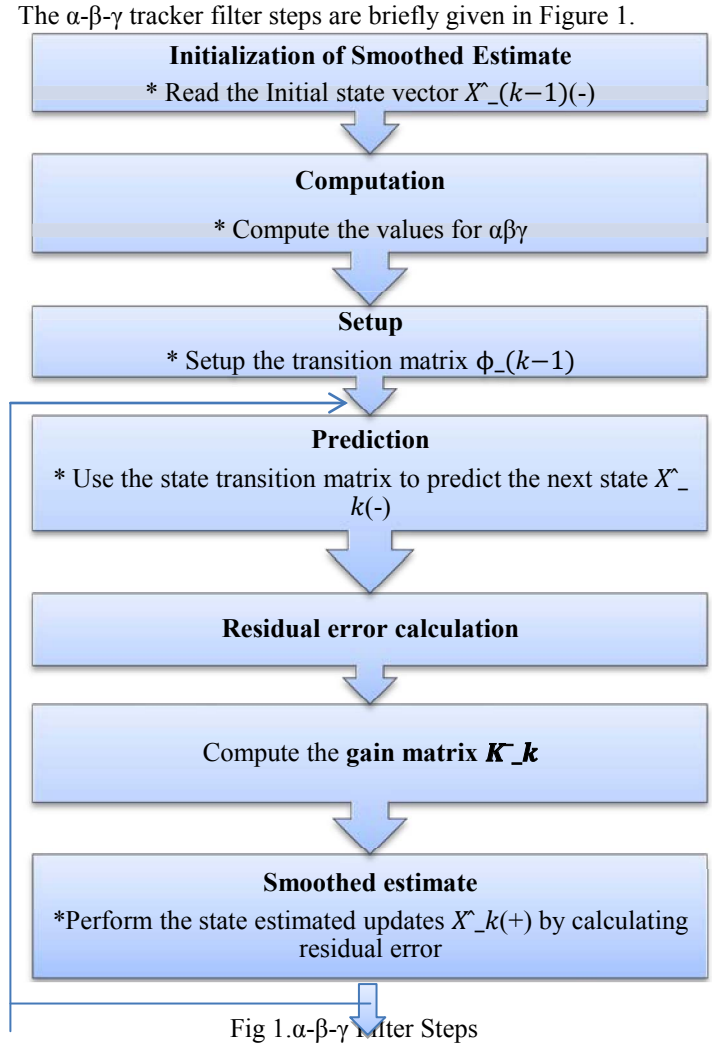

Fig 1.α-β-γ Filter Steps

Now using the Equations from 1 to 9 and by applying the α-β-γ filter steps for the scenario mentioned in section 3 the following simulation results are obtained and it is mentioned in a hierarchal manner shown in the Fig. 2

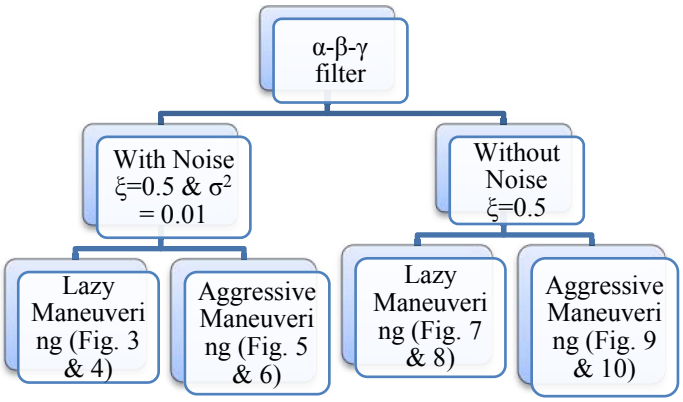

Figure 2.α-β-γ Filter hierarchal structure

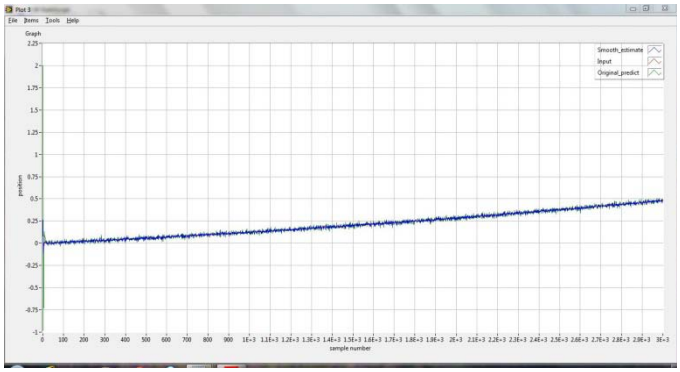

Figure 3 Predicted and True position

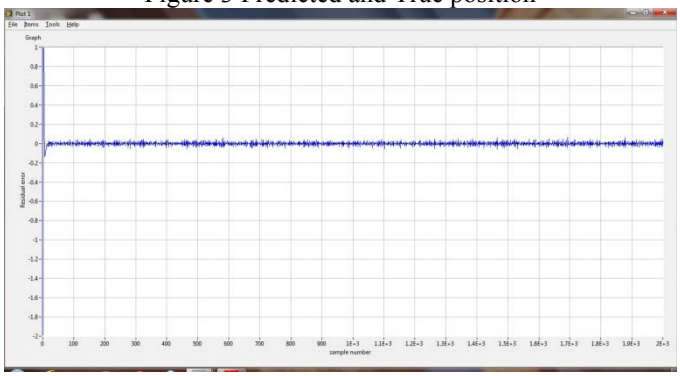

Figure 4 Position Residual error

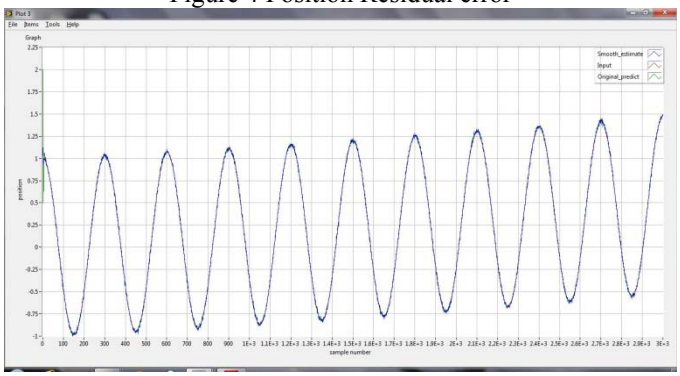

Figure 5 Predicted and True position

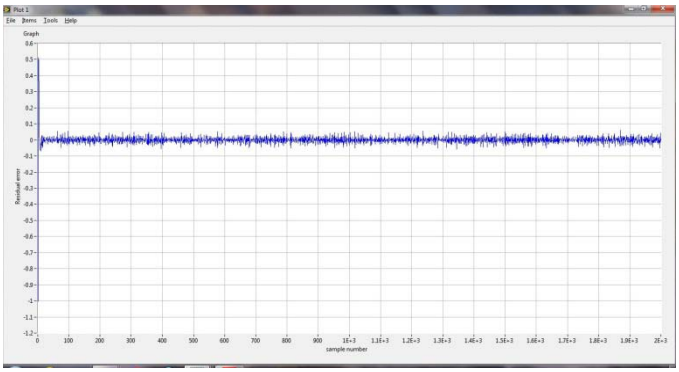

Figure 6 Position Residual error

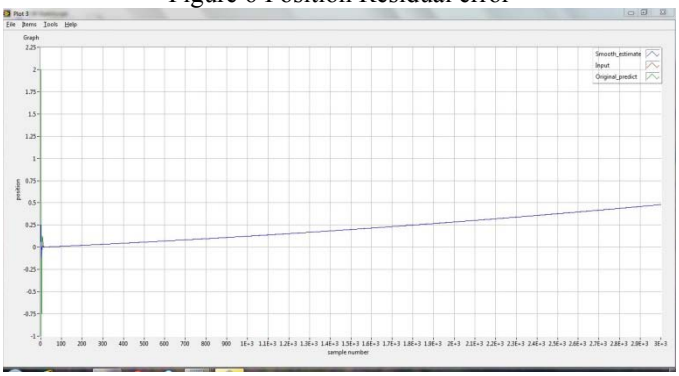

Figure 7 Predicted and True position

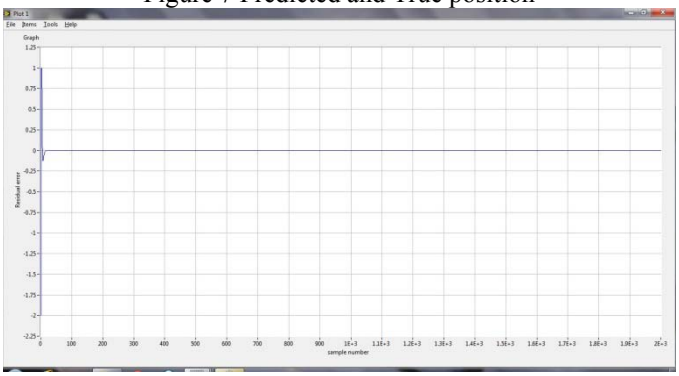

Figure 8 Position Residual error

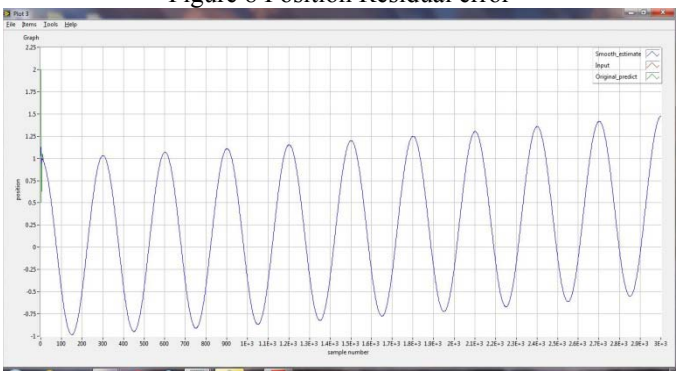

Figure 9 Predicted and True position

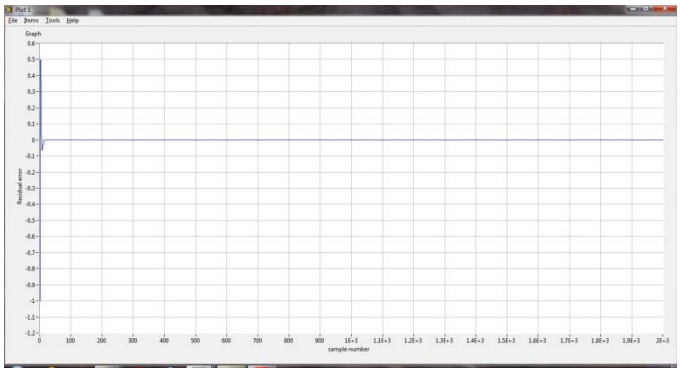

Figure 10 Position Residual error

V. THE KALMAN FILTER APPROACH

Theoretically the Kalman Filter is an estimate for what is called the linear-quadratic problem, which is the problem of estimating the instantaneous "state" of a linear dynamic system perturbed by white noise – by using measurement linearly related to the state but corrupted by white noise. The resultant estimator is statically optimal with respect to any quadratic function of estimation error.Kalman filter gives the best optimal solution for a linear state estimation problem considering the noise in its model as white Gaussian.

A. Linear Quadratic Gaussian Estimation Problem

To derive the mathematical forms of optimal linear estimators for the state of linear stochastic systems is called the Linear Quadratic Gaussian (LQG) estimation problem. The dynamic systems are linear, the performance cost functions are quadratic, and the random process are Gaussian. Filtering, Prediction and Smoothing are three general types of estimators for the LQG problem.

- Predictors use observations strictly prior to the time that the state of the dynamic system is to be estimated:

$$t_{obs} < t_{est}$$

- Filters use observations up to and including the time that the state of the dynamic system is to be estimated:

$$t_{obs} \leq t_{est}$$

- Smoothers use observations beyond the time that the state of the dynamic system is to be estimated:

$$t_{obs} > t_{est}$$

B. Discrete-Time Kalman Filter Equations

- System dynamic model:

$$X_k = \phi_{k-1} X_{k-1} + w_{k-1} \qquad (10)$$

$$w_k \sim N(0, Q_k)$$

- Measurement model:

$$Z_k = H_k X_k + V_k \qquad (11)$$

$$V_k \sim N(0, R_k)$$

- Initial Conditions:

$$E(X_0) = \hat{X}_0 \qquad (12)$$

$$E\left(\tilde{X}_0 \tilde{X}_0^{\,T}\right) = P_0$$

- State estimate extrapolation

$$\hat{X}_k(-) = \phi_{k-1} \hat{X}_{k-1}(+) \qquad (13)$$

- Error covariance extrapolation

$$P_k(-) = \phi_{k-1} P_{k-1}(+) \phi^T{}_{k-1} + Q_{k-1} \qquad (14)$$

- State estimate observational update

$$\hat{X}_k(+) = \hat{X}_k(-) + \bar{K}_k [Z_k - H_k \hat{X}_k(-)] \qquad (15)$$

- Error covariance update

$$P_k(+) = [1 - \bar{K}_k H_k] P_k(-) \qquad (16)$$

- Kalman Gain Matrix

$$\bar{K}_k = P_k(-) H_k{}^T [H_k P_k(-) H_k{}^T + R_k]^{-1} \qquad (17)$$

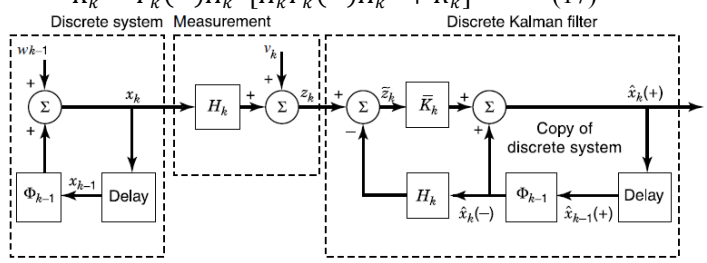

Figure 11.Block diagram of system, measurement model, and discrete-time Kalman Filter

The relation of the filter to the system is illustrated in the block diagram of Figure 1. The basic steps of the computational procedure for the discrete-time Kalman estimator are as follows;

1. Compute $P_k(-)$ using ϕ_{k-1}, $P_{k-1}(+)$ and Q_{k-1}
2. Compute $\bar{K}_k$ using $P_k(-)$, H_k and R_k
3. Compute $P_k(+)$ using $\bar{K}_k$ and $P_k(-)$
4. Compute successive values of $\hat{X}_k(+)$ recursively using computed values of $\bar{K}_k$, given initial estimate $\hat{X}_0$ and the input data z_k [8]

Now using the Equations from 10 to 17 and by applying the Kalmanfilter steps for the scenario mentioned in section 3 the following simulation results are obtained and it is mentioned in a hierarchal manner shown in the Fig. 12. The Kalman filter steps are briefly given in Figure 13 in which the initial values of prediction covariance and transition matrix are taken as follows:

$$P_{k-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \qquad \phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \qquad (18)$$

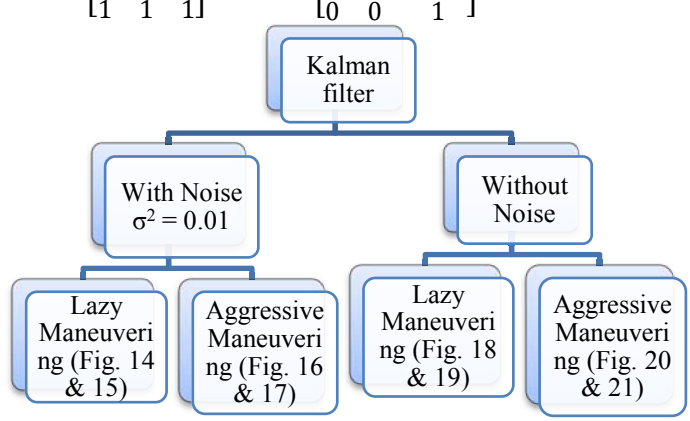

Figure 12.Kalman Filter hierarchal structure

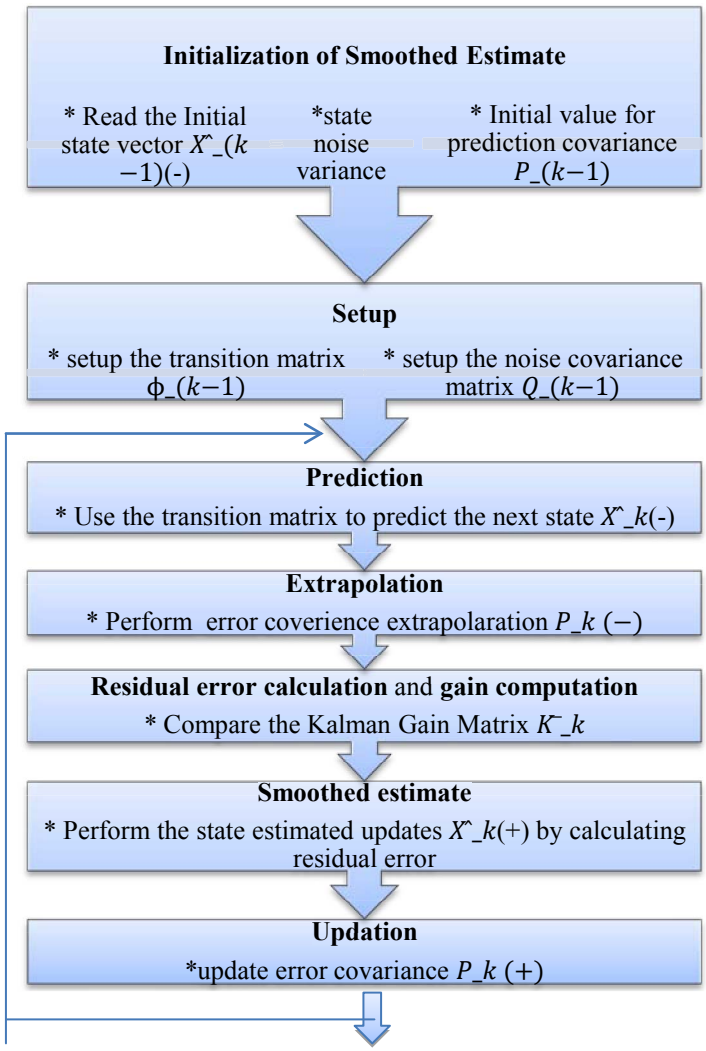

Figure 13Kalman Filter Steps

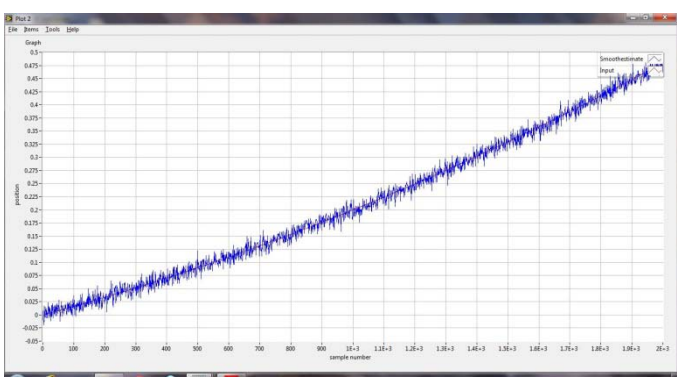

Figure 14 Predicted and True position

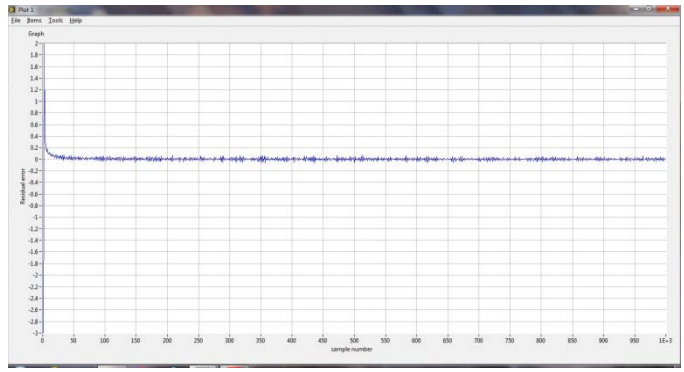

Figure 15 Position Residual error

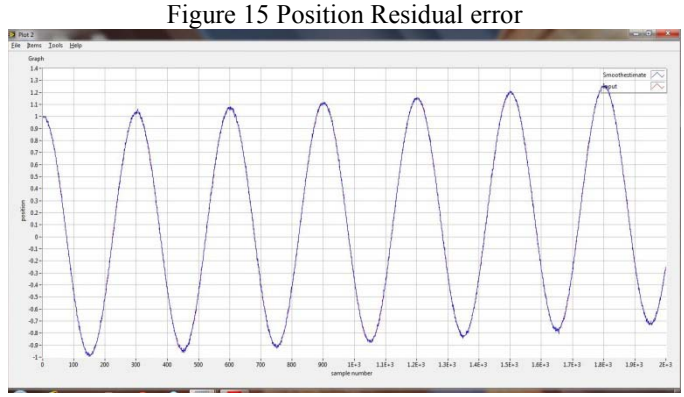

Figure 16 Predicted and True position

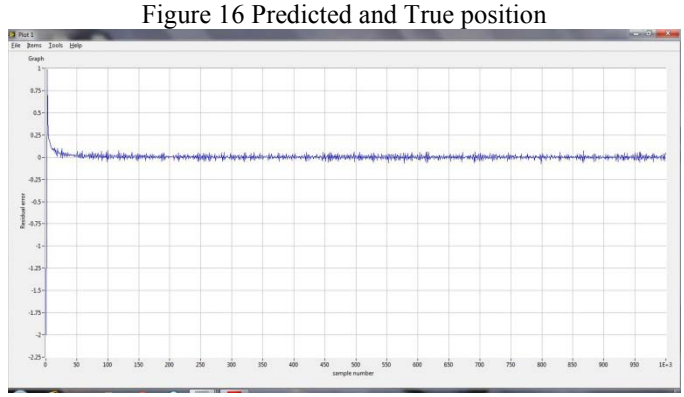

Figure 17 Position Residual error

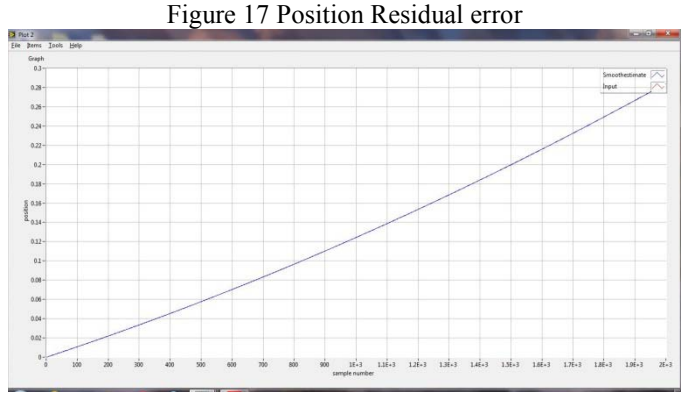

Figure 18 Predicted and True position

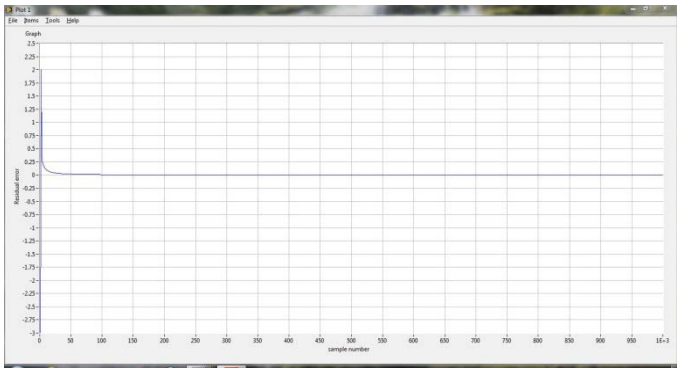

Figure 19 Predicted and True position

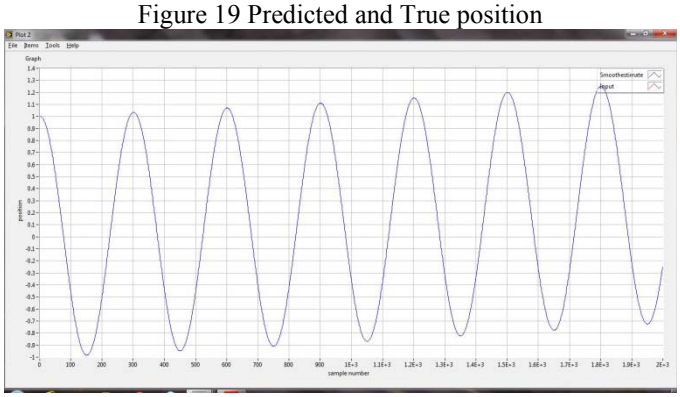

Figure 20 Predicted and True position

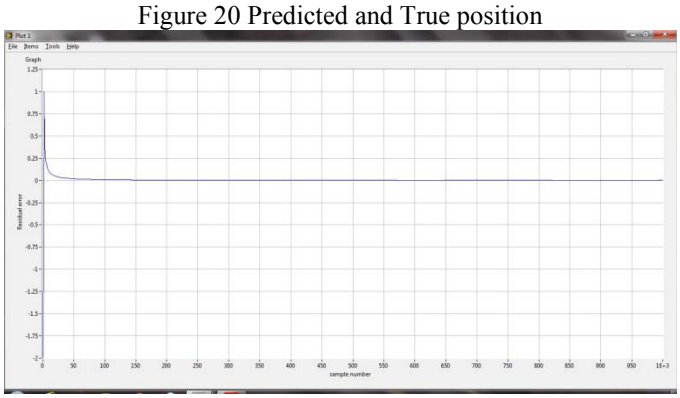

Figure 21 Position Residual error

CONCLUSION

The α-β-γ filter computes gain initially and this gain is further used in smoothing the estimate, and this gain matrix is fixed throughout the process. But this α-β-γ filtercannotspecify the uncertaintylevel.WhereasKalman filter dynamically computes the gain value and the gain matrix is updated in each iteration, and alsoKalman filter describes the uncertainty level at every time step using error covariance matrix.In this paper to estimate the range of the K100 missile which will fly with a relative velocity of 0.32 Km/s, the kalman filter shows the residual error is reduced compared toα-β-γ filter and hence kalman filter gives the best optimal solution for a linear state estimation problem. Kalman filter considerably reduces the noise content in the measurement than that measured by α-β-γ filter.

NOMENCLATURE

X_k- State vector at time k

V_k- Measurement Noise

$f_k/Z_k/h_k$- Non-linear measurement vector

W_k- Process noise

$\underline{X}(t)/\dot{X}(t)$-State vector

$W(t)/Y(t)$-Input vector

X_0-Initial estimate of the state.

$\underline{\phi}$-State estimation matrix

$\hat{X}_k(+)$ - Smoothed estimate at time k

$\hat{X}_k(-)$ - Predicting the state in $(k-1)^{th}$ interval for k^{th} interval

$\underline{K}$-Fixed gain coefficient matrix

$\underline{G}$- Processing noise coupling matrix

αβγ – Fixed gain coefficients

ξ - Smoothing coefficient

T – Sampling time

$\overline{K}_k$- Kalman gain matrix

Q_k-Covariance matrix of state estimation uncertainty

R_k- Covariance matrix of observational uncertainty

$P_k(-)$-Prediction of covariance matrix of state estimation uncertainty

$P_k(-)$- Smoothing of covariance matrix of state estimation uncertainty

H_k-Measurement sensitivity matrix

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