Adaptive Unscented Kalman Filter with Multiple Fading Factors for Pico Satellite Attitude Estimation

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***Abstract*—Thus far, Kalman filter based attitude estimation algorithms have been used in many space applications. When the issue of pico satellite attitude estimation is taken into consideration, general linear approach to Kalman filter becomes insufficient and Extended Kalman Filters (EKF) are the types of filters, which are designed in order to overrun this problem. However, in case of attitude estimation of a pico satellite via magnetometer data, where the nonlinearity degree of both dynamics and measurement models are high, EKF may give inaccurate results. Unscented Kalman Filter (UKF) that does not require linearization phase and so Jacobians can be preferred instead of EKF in such circumstances. Nonetheless, if the UKF is built with an adaptive manner, such that, faulty measurements do not affect attitude estimation process, accurate estimation results even in case of measurement malfunctions can be guaranteed. In this study an Adaptive Unscented Kalman Filter with multiple fading factors based gain correction is introduced and tested on the attitude estimation system of a pico satellite by the use of simulations.**

Keywords-multiple fading factors; adaptive Kalman filter; Uncented Kalman Filter; attitude estimation

I. INTRODUCTION

Since it was proposed, Kalman filter has been widely used as an attitude determination technique [1] and different Kalman filter types have been developed with that purpose. As a known fact; attitude estimation problem of a pico satellite cannot be solved by linear Kalman filters because of the inherent nonlinear dynamics and kinematics. In such case Extended Kalman Filter (EKF) may be used instead. By using EKF, it is possible to estimate attitude parameters of a satellite which has three onboard magnetometers as the only measurement sensors [2]. However, mandatory linearization phase of EKF procedure may cause filter to diverge and usually, Jacobian calculations required for this phase are cumbersome and time-consuming [3, 4].

Unscented Kalman Filter (UKF) is a relatively new Kalman filtering technique that generalizes Kalman filter for both linear and nonlinear systems and in case of nonlinear dynamics, UKF may afford considerably more accurate estimation results than the former observer design methodologies such as Extended Kalman Filter. UKF is based on the fact that; approximation of a nonlinear distribution is easier than the approximation of a nonlinear function or transformation [5]. UKF introduces sigma points to catch higher order statistic of the system and by securing higher order information of the system, it satisfies both, better estimation accuracy and convergence characteristic [6]. Besides, as the estimation characteristic of the UKF is not affected by the level of nonlinearity, it can be preferred for the systems, which has highly nonlinear dynamics and measurements models such as the spacecrafts [7].

On the other hand, UKF has no capability to adapt itself to the changing conditions of the measurement system. Malfunctions such as abnormal measurements, increase in the background noise etc. affects instantaneous filter outputs and process may result with the failure of the filter. In order to avoid from such condition, the filter must be operated adaptively.

UKF can be made adaptive by using various different techniques. Multiple Model Based Adaptive Estimation (MMAE), Innovation Based Adaptive Estimation (IAE) and Residual Based Adaptive Estimation (RAE) are three of basic approaches to the adaptive Kalman filtering. In the first approach, more than one filters run parallel under different models for satisfying filter's true statistical information. However that can be only achieved if the sensor/actuator faults are known. Also, this approach requires several parallel Kalman filters to run and the processing time may increase in such condition [8]. In IAE or RAE methods, adaptation is applied directly to the covariance matrices of the measurement and/or system noises in accordance with the differentiation of the residual or innovation sequence. To realize these methods, the innovation or residual vectors must be known for *m epoch* and that causes an increment in the storage burden, as well as the requirement to know the width of the *moving window* [9]. Besides, in order to estimate covariance matrix of the measurement noise based on the innovation or residual vector; number, type and distribution of measurements must be consistent for all epochs within a window.

Kalman filter may be also built adaptively by using fuzzy logic based techniques. When the theoretical and real innovation values of covariance matrices of the measurement or process noises are compared and a variable, which characterizes the discrepancy level between them, is defined, then by the fuzzy logical rules process or measurement covariance matrices can be adjusted [10, 11]. However, the essences of these kinds of fuzzy methods are human experience and heuristic information; in out of experience cases they may not work.

Another concept is to scale the noise covariance matrix by multiplying it with a time dependent variable. One of the methods for constructing such algorithm is to use a single adaptive factor as a multiplier to the process or measurement noise covariance matrices [8, 12]. This algorithm, which may be named as Adaptive Fading Kalman Filter (AFKF), can be both used when the information about the dynamic or measurement process is absent [13]. Therefore, if there is a malfunction in the measurement system, AFKF algorithm can be utilized and via correction applied to the filter gain, good estimation behaviour of the filter can be secured without being affected from faulty measurements [9].

However, estimation performance of the Kalman filter differs for each variable, when it is utilized for complex systems with multivariable and it may be not sufficient to use single fading factor as a multiplier for the covariance matrices [14]. Single factor may not reflect corrective effects for the faulty measurement to the estimation process, accurately. Technique, which can be implemented to overcome this problem, is to use multiple fading factors (MFF) to fix relevant component of the gain matrix, individually.

Therefore, as an improved methodology for such cases, an AFKF with MFF based filter gain correction may be utilized. In this paper, Adaptive Unscented Fading Kalman Filter (AUFKF) with MFF which combines known UKF and this Adaptive algorithm with multiple fading factors is introduced and applied for attitude parameter estimation of a pico satellite.

The paper proceeds as follow; satellite mathematical model is given in Section II. In the Section III the Earth Magnetic Field and observation models are introduced. The novel AUFKF algorithm is presented in the Section IV. The simulation of testing algorithm as a part of pico satellite attitude parameter estimation procedure is presented in Section V. Section VI gives a brief summary of the obtained results and the conclusion.

II. Satellite Mathematical Model

If the kinematics of the pico satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (φ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame,

$$\bar{x} = \left[\varphi\ \theta\ \psi\ \ \omega_x\ \omega_y\ \omega_z\right]^T , \tag{1}$$

$$\bar{\omega}_{BI} = \left[\ \omega_x\ \omega_y\ \omega_z\ \right]^T, \tag{2}$$

where $\bar{\omega}_{BI}$ is the angular velocity vector of body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$I_x \frac{d\omega_x}{dt} = N_x + \left(I_y - I_z\right)\omega_y\omega_z, \tag{3}$$

$$I_y \frac{d\omega_y}{dt} = N_y + \left(I_z - I_x\right)\omega_z\omega_x, \tag{4}$$

$$I_z \frac{d\omega_z}{dt} = N_z + \left(I_x - I_y\right)\omega_x\omega_y, \tag{5}$$

where I_x, I_y, and I_z are the principal moments of inertia and N_x, N_y, and N_z are the terms of the external moment affecting the satellite. If the gravity gradient torque is taken into the consideration for the Low Earth Orbit (LEO) satellite, these terms can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3}\begin{bmatrix} \left(I_y - I_z\right)A_{23}A_{33} \\ \left(I_z - I_x\right)A_{13}A_{33} \\ \left(I_x - I_y\right)A_{13}A_{23} \end{bmatrix}. \tag{6}$$

Here μ is the gravitational constant, r_0 is the distance between the centre of mass of the satellite and the Earth and A_{ij} represents the corresponding element of the direction cosine matrix of [15].

$$A = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\varphi)s(\psi) + s(\varphi)s(\theta)c(\psi) & c(\varphi)c(\psi) + s(\varphi)s(\theta)s(\psi) & s(\varphi)c(\theta) \\ s(\varphi)s(\psi) + c(\varphi)s(\theta)c(\psi) & -s(\varphi)c(\psi) + c(\varphi)s(\theta)s(\psi) & c(\varphi)c(\theta) \end{bmatrix}. \tag{7}$$

In matrix A, $c(\cdot)$ and $s(\cdot)$ are the cosines and sinus functions successively. Kinematic equations of motion of the pico satellite with the Euler angles can be given as

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)t(\theta) & c(\varphi)t(\theta) \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}. \tag{8}$$

Here $t(\cdot)$ stands for tangent function and p, q, and r are the components of $\bar{\omega}_{BR}$ vector which indicates the angular velocity of the body frame with respect to the reference frame. $\bar{\omega}_{BI}$ and $\bar{\omega}_{BR}$ can be related via,

$$\bar{\omega}_{BR} = \bar{\omega}_{BI} + A\begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix}. \tag{9}$$

where ω_0 denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = (\mu/r_0^3)^{1/2}$.

III. The Earth Magnetic Field Model

As the satellite navigates along its orbit, magnetic field vector differs in a relevant way with the orbital parameters. If those parameters are known, then, magnetic field tensor vector that affects satellite can be shown as a function of time analytically [6, 15]. Note that, these terms are obtained in the orbit reference frame.

$$H_1(t) = \frac{M_e}{r_0^3}\{\cos(\omega_0 t)\left[\cos(\epsilon)\sin(i) - \sin(\epsilon)\cos(i)\cos(\omega_e t)\right] - \sin(\omega_0 t)\sin(\epsilon)\sin(\omega_e t)\} , \tag{10}$$

$$H_2(t) = -\frac{M_e}{r_0^3}\left[\cos(\epsilon)\cos(i) + \sin(\epsilon)\sin(i)\cos(\omega_e t)\right] , \tag{11}$$

$$H_3(t) = \frac{2M_e}{r_0^3}\{\sin(\omega_0 t)\left[\cos(\epsilon)\sin(i) - \sin(\epsilon)\cos(i)\cos(\omega_e t)\right] + 2\sin(\omega_0 t)\sin(\epsilon)\sin(\omega_e t)\} . \tag{12}$$

Here

- $M_e = 7.943\,x\,10^{15} Wb.m$; the magnetic dipole moment of the Earth,
- $\mu = 3.98601\,x\,10^{14} m^3/s^2$; the Earth Gravitational constant,
- $i = 97°$; the orbit inclination,
- $\omega_e = 7.29\,x\,10^{-5} rad/s$; the spin rate of the Earth
- $\epsilon = 11.7°$; the magnetic dipole tilt
- $r_0 = 6{,}928{,}140\ m$; the distance between the centre of mass of the satellite and the Earth

Three onboard magnetometers of pico satellite measures the components of the magnetic field vector in the body frame. Therefore for measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A. Overall measurement model may be given as;

$$\begin{bmatrix} H_x(q,t) \\ H_y(q,t) \\ H_z(q,t) \end{bmatrix} = A \begin{bmatrix} H_1(t) \\ H_2(t) \\ H_3(t) \end{bmatrix}, \tag{13}$$

where, $H_1(t), H_1(t)$, and $H_1(t)$ represents the Earth magnetic field vector components in the orbit frame as a function of time and $H_x(q,t), H_x(q,t)$, and $H_x(q,t)$ shows the Earth magnetic field vector components in body frame as a function of time and varying attitude quaternions.

IV. Adaptive Unscented Fading Kalman Filter For Attitude Estimation

A. *Unscented Kalman Filter*

As it is aforementioned system process model of the satellite which is characterized by dynamic and kinematic equations of motion is nonlinear. That can be easily seen from (2)-(9). Nonetheless, as (13) shows, observation model is also nonlinear with respect to the Euler angles. As a result, in this study UKF is chosen for the estimation of the attitude dynamics parameters $\varphi, \theta, \psi, \omega_x, \omega_y$ and ω_z.

UKF is based on the determination of $2n+1$sigma points with a mean of $\hat{x}(k|k)$ and a covariance of $P(k|k)$ for an n dimensional state vector. These sigma points are obtained by

$$\chi_0(k|k) = \hat{x}(k|k)\,, \tag{14}$$

$$\chi_i(k|k) = \hat{x}(k|k) + \left(\sqrt{(n+\kappa)P(k|k)}\right)_i, \tag{15}$$

$$\chi_{i+n}(k|k) = \hat{x}(k|k) - \left(\sqrt{(n+\kappa)P(k|k)}\right)_i, \tag{16}$$

where, $\chi_0(k|k), \chi_i(k|k)$ and $\chi_{i+n}(k|k)$ are sigma points, n is the state number and κ is the scaling parameter which is used for fine tuning and the heuristic is to choose that parameter as $n+\kappa = 3$ [3].

Next step of the UKF procedure is transforming each sigma point by the use of system dynamics,

$$x_i(k+1|k) = f[x_i(k|k), k]. \tag{17}$$

Then these transformed values are utilized for gaining the predicted mean and the covariance [7].

$$\hat{x}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa\chi_0(k+1|k) + \frac{1}{2}\sum_{i=1}^{2n}\chi_i(k+1|k)\right\} \tag{18}$$

$$\begin{aligned} P(k+1|k) = \tfrac{1}{n+\kappa}\{&\kappa[x_0(k+1|k) - \hat{x}(k+1|k)]\cdot \\ &[x_0(k+1|k) - \hat{x}(k+1|k)] \\ &+\frac{1}{2}\sum_{i=1}^{2n}[x_i(k+1|k) - \hat{x}(k+1|k)]\cdot \\ &[x_i(k+1|k) - \hat{x}(k+1|k)]^T\}\,. \end{aligned} \tag{19}$$

Here, $\hat{x}(k+1|k)$is the predicted mean and $P(k+1|k)$ is the predicted covariance.

Nonetheless, the predicted observation vector is,

$$\hat{y}(k+1|k) = \frac{1}{n+\kappa}\left\{\kappa y_0(k+1|k) + \frac{1}{2}\sum_{i=1}^{2n} y_i(k+1|k)\right\}. \tag{20}$$

After that, observation covariance matrix is determined as,

$$y_i(k+1|k) = h[x_i(k+1|k), v(k), k] \tag{21}$$

$$\begin{aligned} P_{yy}(k+1|k) = \frac{1}{n+\kappa}\{&\kappa[y_0(k+1|k) - \hat{y}(k+1|k)]\cdot \\ &[y_0(k+1|k) - \hat{y}(k+1|k)] \\ &+\frac{1}{2}\sum_{i=1}^{2n}[y_i(k+1|k) - \hat{y}(k+1|k)]\cdot \\ &[y_i(k+1|k) - \hat{y}(k+1|k)]^T\}, \end{aligned} \tag{22}$$

where the innovation covariance is

$$P_{vv}(k+1|k) = P_{yy}(k+1|k) + R(k+1). \tag{23}$$

Here $v(k)$ is the white Gaussian measurement noise and $R(k+1)$ is the measurement noise covariance matrix. On the other hand, the cross correlation matrix can be obtained as,

$$\begin{aligned} P_{xy}(k+1|k) = \frac{1}{n+\kappa}\{&\kappa[x_0(k+1|k) - \hat{x}(k+1|k)]\cdot \\ &[y_0(k+1|k) - \hat{y}(k+1|k)] \\ &+\frac{1}{2}\sum_{i=1}^{2n}[x_i(k+1|k) - \hat{x}(k+1|k)]\cdot \\ &[y_i(k+1|k) - \hat{y}(k+1|k)]^T\}\,. \end{aligned} \tag{24}$$

Following part is the update phase of UKF algorithm. At that phase, first by using measurements, $y(k+1)$, the residual term (or innovation sequence) is found as

$$e(k+1) = y(k+1) - \hat{y}(k+1|k), \tag{25}$$

and Kalman gain is computed via equation of,

$$K(k+1) = P_{xy}(k+1|k)P_{vv}^{-1}(k+1|k). \tag{26}$$

At last, updated states and covariance matrix are determined by,

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)e(k+1)\,, \tag{27}$$

$$P(k+1|k+1) = P(k+1|k) - K(k+1)P_{vv}(k+1|k)K^T(k+1). \tag{28}$$

Here, $\hat{x}(k+1|k+1)$ is the estimated state vector and $P(k+1|k+1)$ is the estimated covariance.

B. *Adaptive Unscented Fading Kalman Filter*

In case of normal operation of the measurement system, UKF works correctly. However, when there is a malfunction in the measurement system such as abnormal measurements, step-like changes or sudden shifts in the measurement channel etc. the filter fails and the estimation outputs become faulty

Hence, an adaptive algorithm must be introduced such that the filter stands robust to the measurement errors (insensitive to failure) and corrects estimations process without affecting good estimation characteristic of the remaining process.

As it is discussed, robustness of the filter may be secured by using single adaptive factor as a corrective term on the filter gain [9]. However that is not a healthy procedure as long as the filter performance differs for each state for the complex systems with multivariable [14]. The preferred method is to use an adaptive matrix built of multiple fading factors to fix the relevant term of the Kalman gain matrix, individually.

Adaptive algorithm affects characteristic of filter only when the condition of the measurement system does not correspond to the model used in the synthesis of the filter. Otherwise filter works with regular algorithm (14)-(28) in an optimal way. In case, where the system operates normally, the real and the theoretical innovation covariance matrix values match as in (29).

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k} e(k+1)e(k+1)^T = P_{yy}(k+1|k)+R(k+1), \quad (29)$$

here, μ is the width of the moving window.

However, when there is a measurement malfunction in the estimation system, the real error will exceed the theoretical one. Hence, if an adaptive matrix, S_k , is added into the algorithm as,

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k} e(k+1)e(k+1)^T = P_{yy}(k+1|k)+S(k)R(k+1), \quad (30)$$

then, it can be determined by the formula of,

$$S_k = \left(\frac{1}{\mu}\sum_{j=k-\mu+1}^{k} e(k+1)e(k+1)^T - P_{yy}(k+1|k)\right)R_k^{-1}. \quad (31)$$

In case of normal operation, the adaptive matrix will be a unit matrix as $S_k = I$. Here I represents the unit matrix.

Nonetheless, as μ is a limited number because of the number of the measurements and the computations performed with computer implies errors such as the approximation errors and the round off errors; $S(k)$ matrix, found by the use of (31) may not be diagonal and may have diagonal elements which are "negative" or lesser than "one" (actually, that is physically impossible).

Therefore, in order to avoid such situation, composing adaptive matrix by the following rule is suggested:

$$S^* = diag(s_1^*, s_2^*, \ldots, s_n^*) \quad (32)$$

where,

$$s_i^* = max\{1, S_{ii}\} \qquad i = 1, n\,. \quad (33)$$

Here, S_{ii} represents the i[th] diagonal element of the matrix S. Apart from that point, if the measurements are faulty, S_k^* will change and so affect the Kalman gain matrix;

$$K(k+1) = P_{xy}(k+1|k)\left[P_{yy}(k+1|k)+S^*(k)R(k+1)\right]^{-1}. \quad (34)$$

In case of any kinds of malfunctions, the related element of the adaptive matrix, which corresponds to the faulty component of the measurement vector, increases and that brings out a smaller Kalman gain, which reduces the effect of the faulty innovation term on the state update process (27). As a result, accurate estimation results can be obtained even in case of measurement malfunctions.

On the other hand, adaptive algorithm is used only in case of faulty measurements and in all other cases procedure is run optimally with regular Unscented Kalman filter. Checkout is satisfied via a kind of statistical information. In order to achieve that, following two hypotheses may be introduced:

- γ_0; the system is normally operating
- γ_1; there is a malfunction in the identification system.

Failure detection is realized by the use of following statistical function,

$$\beta_k = e^T(k+1)\left[P_{yy}(k+1|k)+R(k+1)\right]^{-1}e(k+1). \quad (35)$$

This statistical function has χ^2distribution with s degree of freedom where s is the dimension of the state vector.

If the level of significance, α, is selected as,

$$P\{\chi^2 > \chi^2_{\alpha,M}\} = \alpha; \qquad 0 < \alpha < 1 \quad (36)$$

the threshold value, $\chi^2_{\alpha,s}$ can be determined. Hence, when the hypothesis γ_1 is correct, the statistical value of β_k will be greater than the threshold value $\chi^2_{\alpha,s}$, i.e.:

$$\begin{aligned} \gamma_0 &: \beta_k \le \chi^2_{\alpha,s} && \forall k \\ \gamma_1 &: \beta_k > \chi^2_{\alpha,s} && \exists k\,. \end{aligned} \quad (37)$$

V. SIMULATIONS

Simulations are realized in 20000 steps for a period of 2000 seconds with 0.1 seconds of sampling time, Δt.

During simulations, for testing AUFKF algorithm, two kinds of measurement malfunction scenarios are taken into consideration; instantaneous abnormal measurements, and continuous bias.

Besides, in case of measurement faults, the simulations are also done with UKF so as to compare results with AUFKF and understand efficiency of the adaptive algorithm in a better way.

Nonetheless, $\chi^2_{\alpha,s}$ is taken as 12.592 and this value comes from chi-square distribution when the degree of freedom is 6 and the reliability level is %95.

First part of figures gives UKF or AUFKF state estimation results and the actual values in a comparing way. Second part of the figures shows the error of estimation process based on the actual attitude estimation values of the satellite. The last part indicates the variance of the estimation.

A. Instantenous Abnormal Measurements

Instantaneous abnormal measurements are simulated by adding a constant term to the magnetic field tensor measurement of one magnetometer at the 500th second. As it is seen from Fig. 1 and Fig. 2, AUFKF algorithm with MFF based gain correction gives more accurate estimation results in case of the instantaneous abnormal measurements. The results obtained by regular UKF are not reliable when the measurements are gained with an error. However, AUFKF maintains its estimation characteristic for the whole process and affords precise estimation outputs in case of the abnormal measurements, as well as the normal operation condition. Similar results have been obtained when the measurement malfunction is implemented to another magnetometer.

Table I compares absolute estimation errors of both filters for two different time steps. Note that, highlighted results are gained at seconds where the measurement malfunction is implemented.

As a result of the dynamics and the measurement models of the satellite which are nonlinear and constituted of coupled states, any malfunction in one of the magnetometers affects the estimation characteristic of all states, but not in same degree. Hence, for this kind of measurement malfunction, AUFKF with MFF proves its high performance capability as it can correct estimation characteristic of each state, individually and secure accurate estimation characteristic for all states at the same time.

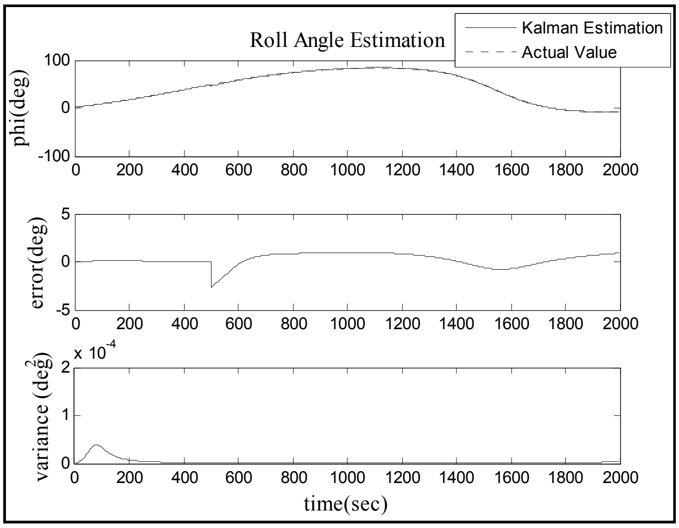

Figure 1. Roll angle estimation with regular UKF in case of instantaneous abnormal measurements.

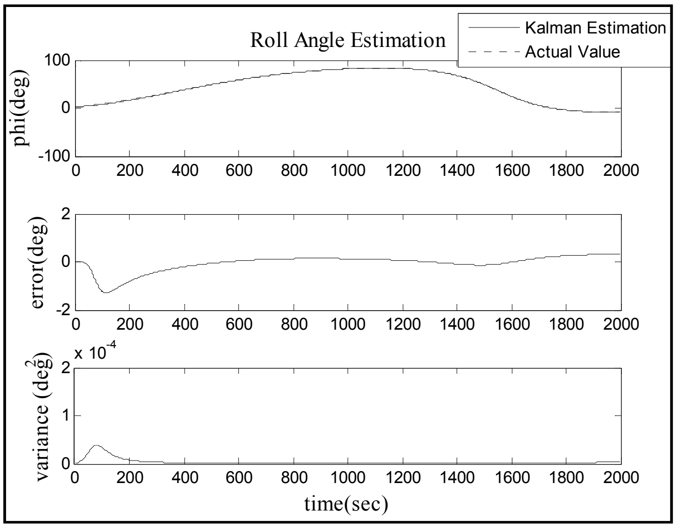

Figure 2. Roll angle estimation with AUFKF with MFF in case of instantaneous abnormal measurements.

TABLE I. COMPARISON OF ABSOLUTE ESTIMATION ERRORS IN CASE OF INSTANTANEOUS ABNORMAL MEASUREMENTS.

Parameter	Abs. Err. Values for Regular UKF		Abs. Err. Values for AUFKF with MFF	
	500 s.	1000 s.	500 s.	1000 s.
$\varphi\ (deg)$	2.6941	0.9987	0.0652	0.1286
$\theta\ (deg)$	1.1066	0.8125	0.3746	0.0705
$\psi\ (deg)$	0.7437	5.1317	0.7132	0.9247
$\omega_x\ (deg/s)$	0.028	0.002	0.0003	0.0003
$\omega_y\ (deg/s)$	0.0181	0.0055	0.0007	0.0007
$\omega_z\ (deg/s)$	0.0007	0.0005	0.0013	0.00004

B. Continuous Bias at Measurements

Continuous bias term is formed by adding a constant term to the measurements of one of the magnetometer in between 500th and 530th seconds. As Fig. 3, Fig. 4 and Table II show, again regular UKF fails about estimating states accurately. Per contra, AUFKF algorithm reduces the effect of the innovation sequence and eliminates the estimation error which is caused by the biased measurements of one magnetometer. Besides, by the use of the predicted states which are more weighted than the innovation sequence in (27), it secures accurate estimation outputs throughout this period.

TABLE II. COMPARISON OF ABSOLUTE ESTIMATION ERRORS IN CASE OF CONTINUOUS BIAS AT MEASUREMENTS

Parameter	Abs. Err. Values for Regular UKF		Abs. Err. Values for AUFKF with MFF	
	500 s.	1000 s.	500 s.	1000 s.
$\varphi\ (deg)$	5.0535	15.759	0.0928	0.2329
$\theta\ (deg)$	2.7733	23.431	0.6617	0.1156
$\psi\ (deg)$	1.2492	192.76	1.256	1.6444
$\omega_x\ (deg/s)$	0.0558	0.0798	0.0005	0.0006
$\omega_y\ (deg/s)$	0.0352	0.2307	0.0013	0.0013
$\omega_z\ (deg/s)$	0.0043	0.1422	0.0021	0.0001

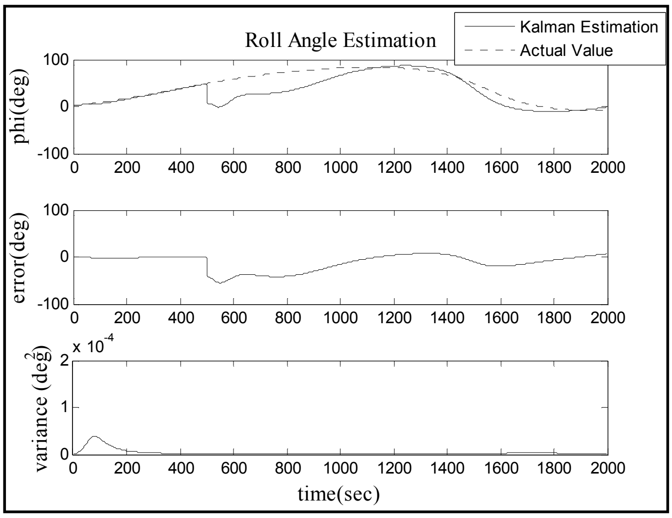

Figure 3. Roll angle estimation with regular UKF in case of continuous bias at measurements.

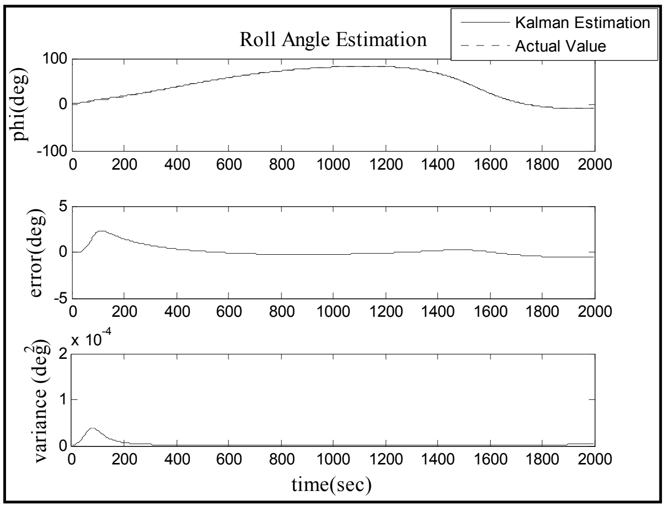

Figure 4. Roll angle estimation with AUFKF with MFF in case of continuous bias at measurements.

VI. Conclusion

In this paper an Adaptive Uncented Fading Kalman Filter with Multiple Fading Factors for the case of measurement malfunctions are developed. By the use of defined variables named as fading factor, faulty measurements are taken into consideration with small weight and the estimations are corrected without affecting the characteristic of the accurate ones. In the presented AUFKF the filter gain correction is performed only in the case of malfunctions in the measurement system.

Proposed AUFKF algorithm with MFF is applied for the state estimation process of a pico satellite's attitude dynamics model. Algorithm is tested for two different measurement malfunction scenarios and results are compared with the outputs of UKF for the same cases; Instantaneous abnormal measurements and continuous bias at measurements. At both circumstances UKF becomes faulty while the introduced AUFKF algorithm stands robust to the measurement errors (insensitive to failure). Comparison of simulation results show that, the performance of AUFKF with MFF is significantly better than UKF in the case of measurement malfunctions.

The proposed approach does not require a priori statistical characteristics of the faults. Furthermore the presented AUFKF algorithm with MFF is simple for practical implementation and its computational burden is not heavy. These characteristics make introduced AUFKF algorithm extremely important in point of view of supplying reliable state estimation for the attitude determination and control system of pico satellites.

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