

# An Interval Fuzzy Controller for Vehicle Active Suspension Systems

Jiangtao Cao, *Member, IEEE*, Ping Li, *Senior Member, IEEE*, and Honghai Liu, *Senior Member, IEEE*

**Abstract**—A novel interval type-2 fuzzy controller architecture is proposed to resolve nonlinear control problems of vehicle active suspension systems. It integrates the Takagi–Sugeno (T–S) fuzzy model, interval type-2 fuzzy reasoning, the Wu–Mendel uncertainty bound method, and selected optimization algorithms together to construct the switching routes between generated linear model control surfaces. The stability analysis of the proposed approach is presented. The proposed method is implemented into a numerical example and a case study on a nonlinear half-vehicle active suspension system. The simulation results demonstrate the effectiveness and efficiency of the proposed approach.

**Index Terms**—Active suspension system, interval type-2 fuzzy control, stability analysis.

## I. INTRODUCTION

VEHICLE suspension systems, as a kind of typical nonlinear system, play a crucial role in riding comfort, safety handling, and road-damage minimization and significantly contribute to the overall vehicle performance. It is evident that tradeoffs have to be taken to achieve an overall better performance for all types of suspension systems, including passive, semiactive, and active suspensions [1]. Increasing attention has been paid to active suspensions in recent years, mainly due to its less-physical constraints, flexible structure, and intelligent methodology to deal with random vibrations. It is also evident that different developing control algorithms of the core part of active suspension systems have significantly contributed in improving suspension performance [2], [3].

To control an active suspension, the control algorithms must be able to deal with mechanical nonlinear dynamics and be operated under imprecise and uncertain conditions, which are mainly caused by random natural road surfaces. The mechanism behind fuzzy logic controllers (FLCs) is credited with being a feasible methodology for designing robust controllers that are able to deliver satisfying performance in the face of nonlinearity, uncertainty, and imprecision [4]. Hence, FLCs

have become a popular approach for active suspension systems in recent years. There are different ways to construct FLCs for vehicle suspension control systems. It is common to construct an FLC by eliciting the corresponding fuzzy rules and defining their membership functions based on expert knowledge or industrial experience. More importantly, contributions have been made to improve the basic fuzzy control structure with computational intelligent strategies. It is evident that integrating the fuzzy controllers with other intelligent methods such as neural networks and genetic algorithms improve control performance in uncertain scenarios [5]–[11].

From the viewpoint of application, all existing suspension control systems employ type-1 fuzzy sets to build a type-1 fuzzy control system. It limits introducing uncertain factors from linguistic rules through predefined membership functions. To overcome the weakness, type-2 fuzzy sets have recently been proposed with their more general fuzzy membership functions and potential ability to solve real-world uncertain scenarios [12]–[14]. The concept of the type-2 fuzzy set was originally introduced by Zadeh [15] as an extension of the ordinary fuzzy set. Then, it has significantly been developed from theoretical research to applications in the past decade. A fuzzy logic system consisting of at least one type-2 fuzzy set is called a type-2 FLS [16]. In comparison with the type-1 FLS, a type-2 FLS has twofold advantages as follows: First, it has the capability of directly handling the uncertain factors of fuzzy rules caused by expert experience or linguistic description. Second, it is efficient to employ a type-2 FLS to cope with scenarios in which it is difficult or impossible to determine an exact membership function and related measurement of uncertainties. These strengths have made researchers consider type-2 FLS as the preference for real-world applications.

The interval type-2 fuzzy system (IT2 FS) is one of the main branches of type-2 fuzzy systems. It has widely been studied and utilized for real-time control systems mainly due to its lower computational cost [3], [17]–[19]. There are many types of methods for IT2 FSs to aggregate IT2 fuzzy values to crisp outputs. Here, two of them are considered: One is represented by the Karnik–Mendel (K–M) algorithm, which involves type reduction (TR) [20], [21], and the other is represented by Wu–Mendel uncertainty bounds, which do not involve TR [18]. The first type of method calculates the exact solutions monotonically and super exponentially fast with a simple formula; however, the time delay caused by algorithmic iteration is the bottleneck for real-time applications. On the other hand, the second type of method replaces the TR by four uncertainty bounds. These bounds only depend on the lower and upper firing levels of each rule and the centroid of each rule's

Manuscript received January 8, 2009; revised January 19, 2010; accepted June 5, 2010. Date of publication July 8, 2010; date of current version December 3, 2010. This work was supported in part by the Program for Liaoning Excellent Talents in University under Grant 2008RC32 and in part by the Program for Liaoning Science and Technology Innovative Research Team in University under Grant LT2010058. The Associate Editor for this paper was U. Nunes.

J. Cao and P. Li are with the School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China (e-mail: jtcao@lnpu.edu.cn; liping@lnpu.edu.cn).

H. Liu is with the School of Creative Technologies, University of Portsmouth, PO1 2TG Portsmouth, U.K. (e-mail: Honghai.liu@port.ac.uk).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TITS.2010.2053358

consequent set. Wu and Mendel [18] calculated the final crisp outputs of an IT2 FLS by averaging the uncertainty bounds. That means that it is computationally efficient that the mean operator and four uncertainty bounds are combined to estimate their defuzzified outputs. It is obvious that existing aggregation operators can play the mean method's role in defuzzification [16] to achieve better performance for individual applications. Some new researches on defuzzification of type-2 fuzzy sets are presented in [22] and [23]

Inspired by Mendel's work [16], this paper proposed a general structure to aggregate uncertainty bounds to the defuzzified outputs through a further-optimization structure. It integrates the Takagi–Sugeno (T–S) fuzzy model, the interval type-2 fuzzy reasoning, the Wu–Mendel uncertainty bounds, and selected optimization algorithms together to construct the switching routes among the generated linear model control surfaces. Considering the uncertainty bounds and further-optimization algorithms, based on the common quadratic Lyapunov functions, the stability analysis of the closed-loop control system is presented. By integrating the T–S fuzzy model into the proposed architecture of the IT2 FLS, the further-optimization module rebuilds the transfer routes between the generated linear control surfaces and integrates the control performance and other practical requirements into the defuzzification interface. For evaluation purposes, the proposed structure is implemented into a numerical example and a case study on a half-vehicle active suspension system with convincing results.

The rest of this paper is organized as follows: Section II presents a nonlinear model of a half-vehicle active suspension system. Section III proposes an IT2 T–S fuzzy control system with a further-optimization structure. Section IV analyzes the stability of the closed-loop control system with the proposed architecture. Some simulations are given in Section V. Concluding remarks and future work are discussed in Section VI.

## II. NONLINEAR ACTIVE SUSPENSION SYSTEM

A vehicle body is generally a rigid body with 6 degree-of-freedom (DOF) motions and consists of longitudinal, lateral, and heave motions, as well as roll, pitch, and yaw motions [2]. These motions are restricted by the geometrical constraints of a vehicle suspension and also coupled with each other to a certain degree. Regardless of such coupling problems, the reduced-order mathematical model is commonly employed to design an active suspension control system. Therefore, a quarter-vehicle model or a half-vehicle model is often used for theoretical analysis and design of active suspension systems.

A half-vehicle model including pitch and heave modes is represented to simulate the ride characteristics of a simplified whole vehicle. Based on the half-vehicle model, many active control strategies have been designed to improve the ride comfort and handling performance of vehicle suspensions [24]. Let  $f$  and  $r$  denote the front and rear wheels and  $x$  and  $z$  denote the longitudinal forward and vertical up directions. The notation is provided here for the half-vehicle model, as shown in Fig. 1.

$d_f$  Distance from the front axle to the center of gravity (in meters).

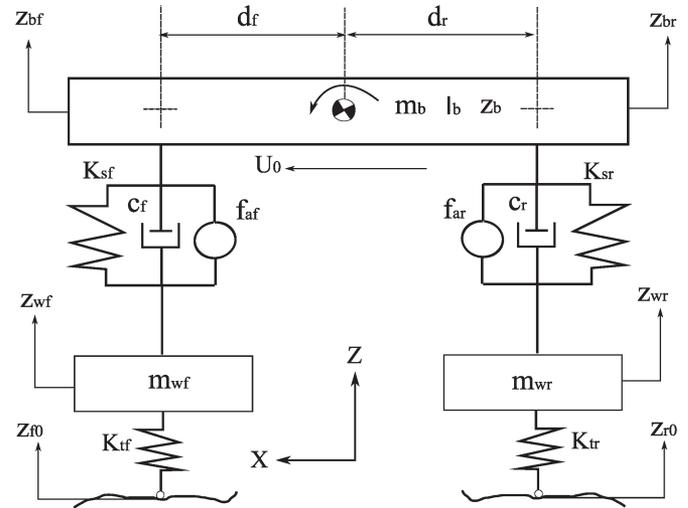


Fig. 1. Half-vehicle suspension model in [24].

- $d_r$  Distance from the rear axle to the center of gravity (in meters).
- $m_b$  Half body mass (or sprung mass) (in kilograms).
- $m_{wf}$  Front wheel mass (or unsprung mass) (in kilograms).
- $m_{wr}$  Rear wheel mass (or unsprung mass) (in kilograms).
- $K_{sf}$  Front suspension spring stiffness (in Newton per meter).
- $K_{sr}$  Rear suspension spring stiffness (in Newton per meter).
- $K_{tf}$  Front tire stiffness (in Newton per meter).
- $K_{tr}$  Rear tire stiffness (in Newton per meter).
- $c_f$  Front damping coefficient (in Newton-second per meter).
- $c_r$  Rear damping coefficient (in Newton-second per meter);
- $f_{af}$  Front actuator force (in Newton);
- $f_{ar}$  Rear actuator force (in Newton);
- $U_0$  Original velocity (in meters per second);
- $I_b$  Pitch inertia (in square kilograms).
- $z_{f0}$  Road displacement at the front wheel (in meters).
- $z_{r0}$  Road displacement at the rear wheel (in meters).
- $z_{wf}$  Front wheel displacement (in meters).
- $z_{bf}$  Front body displacement (in meters).
- $z_{wr}$  Rear wheel displacement (in meters).
- $z_{br}$  Rear body displacement (in meters).

As Hrovat remarked in [1], the linear system approximation was appropriate for some operations; however, there were some situations that amplified the nonlinear effects. These nonlinear effects could be created by dry friction and discrete-event disturbances (e.g., single bumps or potholes). To simulate the real suspension system and to evaluate the potentially application of the proposed control method, a nonlinear model is better to precisely describe the real system dynamics than linear models. Meanwhile, for vehicle suspension systems, the high-order polynomial functions are better than the proportional functions to describe the real spring and damper forces. Based on the method in [25], the connecting forces (e.g., spring force and damping force) can be modeled as the nonlinear functions using measured data. The spring force  $f_s$  is estimated by high-order polynomial functions, as shown in

$$f_s = f_{s1} + f_{s2} = k_1 \Delta z + (k_0 + k_2 \Delta z^2 + k_3 \Delta z^3) \quad (1)$$

where  $f_{sl}$  is the linear term of the spring force, and  $f_{sn}$  is the nonlinear term of the spring force. The coefficients are obtained by fitting the equation to experimental data. The damping force  $f_d$  is also modeled as a second-order polynomial function by fitting the measured data. It is given as follows:

$$f_d = f_{dl} + f_{dn} = c_1 \Delta \dot{z} + c_2 \Delta \dot{z}^2 \quad (2)$$

where  $f_{dl}$  is the linear term, and  $f_{dn}$  is the nonlinear term of the damper force. Likewise, the coefficients are obtained by the measured data fitting.

In addition to the nonlinear properties presented by the spring force and damping force, the vertical tire force is also highly nonlinear, particularly when there are substantial load changes. Even the vertical tire force becomes zero when the tire loses contact with the road surface. The tire force is modeled as follows:

$$\begin{aligned} f_{tl} &= k_t(z_0 - z_w), & \text{when } (z_0 - z_w) > 0 \\ f_{tn} &= 0, & \text{when } (z_0 - z_w) \leq 0 \end{aligned}$$

where  $f_{tl}$  denotes the linear tire force, and  $f_{tn}$  denotes the nonlinear tire force.

The nonlinear model of an active suspension system is provided here for controller design and performance analysis. Considering the nonlinearity shown in (1) and (2), the active suspension system can be written as a multiple-input–multiple-output nonlinear model

$$\dot{X} = F(X, U) \quad (3)$$

where  $X$  denotes the state matrices, which include the displacements and velocity of the vehicle body (i.e.,  $\dot{Z}_b$  and  $Z_b$ ), suspension (i.e.,  $\dot{Z}_w$  and  $Z_w$ ), and road surface input (i.e.,  $\dot{Z}_0$  and  $Z_0$ );  $U$  denotes the actuator force matrices (i.e.,  $f_{af}$  and  $f_{ar}$ ); and  $F(X, U)$  is a nonlinear function that presents the suspension nonlinear dynamic description and can be obtained by integrating the linear model in [3] and the nonlinear forces (i.e.,  $f_s$ ,  $f_t$ , and  $f_d$ ).

### III. INTERVAL TYPE-2 TAKAGI–SUGENO FUZZY CONTROL SYSTEM

Extensive studies, particularly fuzzy control strategies, have been conducted to overcome the nonlinearity and uncertainty of active suspension systems. A brief introduction on general T–S fuzzy control is first presented in this section. Then, the interval membership functions, type-2 reasoning methods, and proposed optimization structure are demonstrated. Finally, the section is concluded with a novel IT2 T–S fuzzy control system.

#### A. General T–S Fuzzy Model and Fuzzy Control System

Considering a T–S fuzzy model, it is represented as the general form

$$\begin{aligned} R^{(l)} : & \text{ IF } z_1 \text{ is } F_1^l \text{ and } z_2 \text{ is } F_2^l, \dots, \text{ and } z_\nu \text{ is } F_\nu^l \\ & \text{ THEN } x(t+1) \text{ is } g^l(X, U) \\ & \text{ where } l \in L := 1, 2, \dots, m \end{aligned} \quad (4)$$

where  $R^{(l)}$  denotes the  $l$ th fuzzy inference rule,  $m$  denotes the number of fuzzy rules,  $F_j^l$  ( $j = 1, 2, \dots, \nu$ ) denotes the type-1 fuzzy sets,  $z(t) := [z_1, z_2, \dots, z_\nu]$  denote measurable variables,  $x(t) \in \mathbb{R}^n$  denotes the state vector,  $u(t) \in \mathbb{R}^p$  denotes the input vector, and the T–S consequent terms  $g_i^l$  is defined in

$$g^l(X, U\theta^l) = A_l x(t) + B_l u(t) + a_l, \quad l \in L := 1, 2, \dots, m \quad (5)$$

where  $A_l$ ,  $B_l$ , and  $a_l$  are the parameter matrices of the  $l$ th local model.

The fuzzy control scheme is chosen as the parallel distributed compensation control, and it is defined as follows:

$$\begin{aligned} R^{(r)} : & \text{ IF } z_1 \text{ is } F_1^r \text{ and } z_2 \text{ is } F_2^r, \dots, \text{ and } z_\nu \text{ is } F_\nu^r \\ & \text{ THEN } u(t) \text{ is } K_r x(t), \quad r \in L := 1, 2, \dots, m \end{aligned} \quad (6)$$

where  $K_r$  stands for the  $r$ th local linear control gain.

By using a singleton fuzzifier, product inference operator, and center average defuzzifier, with the affine terms  $a_l \equiv 0$ , the closed-loop fuzzy control system can be rewritten as

$$x(t+1) = \sum_{l=1}^m \sum_{r=1}^m \mu_l \mu_r (A_l + B_l K_r) x(t). \quad (7)$$

It is assumed that  $\mu_l$  and  $\mu_r$ , which are the normalized membership functions, both satisfy the following:

$$\begin{aligned} \mu_l &= \frac{\xi_l(z)}{\sum_{i=1}^m \xi_i(z)} \\ \xi_l(z) &= \prod_{i=1}^{\nu} F_i^l(z_i) \\ \mu_l &\geq 0, \quad \sum_{l=1}^m \mu_l = 1 \end{aligned} \quad (8)$$

where  $F_i^l(z_i)$  is the membership grade of  $z_i$  in fuzzy set  $F_i^l$ .

#### B. Interval Type-2 T–S Fuzzy Control System

Although the priority in real-time applications has been given to the IT2 FLS in that it has the capability of handling higher order uncertainty factors in terms of cheaper computational cost and simple structure, it has difficulties in interpreting related uncertainty scenarios in the IT2 fuzzy reasoning. Hence, an IT2 T–S fuzzy control system is presented in this section to analyze the problem of how to bound the potential uncertainty in the type-2 reasoning process.

To present the general structure of the IT2 T–S FLS, the  $l$ th rule in (4) can be rewritten as the following format:

$$\begin{aligned} R^{(l)} : & \text{ IF } z_1 \text{ is } \tilde{F}_1^l \text{ and } z_2 \text{ is } \tilde{F}_2^l, \dots, \text{ and } z_\nu \text{ is } \tilde{F}_\nu^l \\ & \text{ THEN } x(t+1) = A_l x(t) + B_l u(t) \\ & (l \in L := 1, 2, \dots, m) \end{aligned} \quad (9)$$

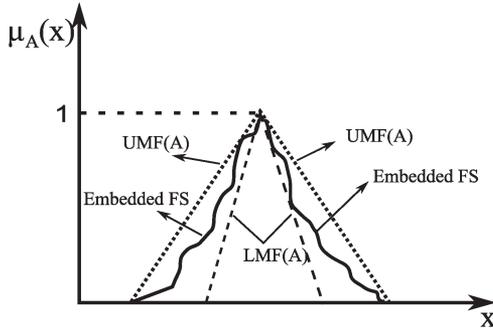


Fig. 2. Example of an IT2 fuzzy membership function, where UMF stands for upper membership function, and LMF stands for lower membership function [19].

where  $\tilde{F}_i^l$  is an interval type-2 fuzzy set of rule  $l$ , which corresponds to a membership function shown in Fig. 2. Its firing strength of the  $l$ th rule belongs to the following interval set:

$$\omega_l(x) \in [\underline{\omega}_l(x), \bar{\omega}_l(x)], \quad l = 1, 2, \dots, m \quad (10)$$

where

$$\underline{\omega}_l(x) = \underline{\mu}_{\tilde{F}_1^l}(x) * \underline{\mu}_{\tilde{F}_2^l}(x) * \dots * \underline{\mu}_{\tilde{F}_m^l}(x) \quad (11)$$

$$\bar{\omega}_l(x) = \bar{\mu}_{\tilde{F}_1^l}(x) * \bar{\mu}_{\tilde{F}_2^l}(x) * \dots * \bar{\mu}_{\tilde{F}_m^l}(x) \quad (12)$$

in which  $\underline{\mu}_{\tilde{F}_i^l}(x)$  and  $\bar{\mu}_{\tilde{F}_i^l}(x)$  denote the lower and upper membership grades, respectively. Then, the inferred IT2 T-S fuzzy model is defined as

$$\begin{aligned} x(t+1) &= \sum_{l=1}^m (\alpha \cdot \underline{\omega}_l(x) + \beta \cdot \bar{\omega}_l(x)) (A_l x + B_l u) \\ &= \sum_{l=1}^m \tilde{\omega}_l(x) (A_l x + B_l u) \end{aligned} \quad (13)$$

where

$$\begin{aligned} \tilde{\omega}_l(x) &= \alpha \cdot \underline{\omega}_l(x) + \beta \cdot \bar{\omega}_l(x) \in [0, 1] \\ \sum_{l=1}^m \tilde{\omega}_l(x) &= 1. \end{aligned} \quad (14)$$

Herein, the values of  $\alpha$  and  $\beta$  are both set as 0.5, according to [26].

To control a nonlinear plant based on the IT2 T-S fuzzy model described by (13), an IT2 T-S fuzzy controller is designed, and its fuzzy rules are given as follows:

$$\begin{aligned} R^{(r)} : \quad &\text{IF } z_1 \text{ is } \tilde{F}_1^r \text{ and } z_2 \text{ is } \tilde{F}_2^r, \dots, \text{ and } z_\nu \text{ is } \tilde{F}_\nu^r \\ &\text{THEN } u(t) \text{ is } \tilde{K}_r x(t) \quad (r \in L := 1, 2, \dots, m) \end{aligned} \quad (15)$$

where  $\tilde{K}_r$  stands for the  $r$ th local linear control gain. The output of this controller is defined as

$$u(t) = \sum_{r=1}^m f(\omega_r^L(x), \omega_r^U(x)) \tilde{K}_r \cdot x \quad (16)$$

where

$$\omega_r^L(x) = \frac{\underline{\omega}_r(x)}{\sum_{r=1}^m (\underline{\omega}_r(x) + \bar{\omega}_r(x))} \quad (17)$$

$$\omega_r^U(x) = \frac{\bar{\omega}_r(x)}{\sum_{r=1}^m (\underline{\omega}_r(x) + \bar{\omega}_r(x))}. \quad (18)$$

$\omega_r^L$  and  $\omega_r^U$  are satisfied with

$$\sum_{r=1}^m (\omega_r^L(x) + \omega_r^U(x)) = 1 \quad (19)$$

and the value of  $f(\omega_r^L(x), \omega_r^U(x))$  depends on the TR methods and belongs to an interval.

The TR method is employed in this section and based on minimax uncertainty bounds [16], [18]. Let us assign  $(\omega_r^L(x) + \omega_r^U(x))/2$  to  $f(\omega_r^L(x), \omega_r^U(x))$  and substitute it into (16); we obtain the following:

$$u(t) \in [u^{(O)}(t), u^{(M)}(t)] \quad (20)$$

where

$$u^{(O)}(t) = \frac{\sum_{i=1}^m \underline{\omega}^i K_i x}{\sum_{i=1}^m \underline{\omega}^i} \quad (21a)$$

$$u^{(M)}(t) = \frac{\sum_{i=1}^m \bar{\omega}^i K_i x}{\sum_{i=1}^m \bar{\omega}^i}. \quad (21b)$$

Then, the uncertainty bounds can be calculated by

$$\bar{u}_c(t) = \min \{u^{(O)}(t), u^{(M)}(t)\} \quad (22a)$$

$$\begin{aligned} \underline{u}_c(t) &= \bar{u}_c(t) - \left[ \frac{\sum_{i=1}^m (\bar{\omega}^i - \underline{\omega}^i)}{\sum_{i=1}^m \bar{\omega}^i \sum_{i=1}^m \underline{\omega}^i} \right. \\ &\quad \times \left. \frac{\sum_{i=1}^m \underline{\omega}^i (K_i - K_1) x \sum_{i=1}^m \bar{\omega}^i (K_m - K_i) x}{\sum_{i=1}^m \underline{\omega}^i (K_i - K_1) x + \sum_{i=1}^m \bar{\omega}^i (K_m - K_i) x} \right]. \end{aligned} \quad (22b)$$

The lower bound  $\underline{u}_c(x)$  is assigned to be equal to the upper bound  $\bar{u}_c(x)$  if only one rule is fired (i.e.,  $m = 1$ ).

The crisp output of the controller is

$$u(t) \approx \frac{1}{2} [\underline{u}_c(t) + \bar{u}_c(t)]. \quad (23)$$

Generally speaking, the T-S fuzzy model is employed to describe a global nonlinear system in terms of a set of local linear models that are smoothly connected by fuzzy membership functions. Herein, the IT2 T-S fuzzy model supplies an alternative way to build the bounded interval switching routes among local linear models, which means that, under the

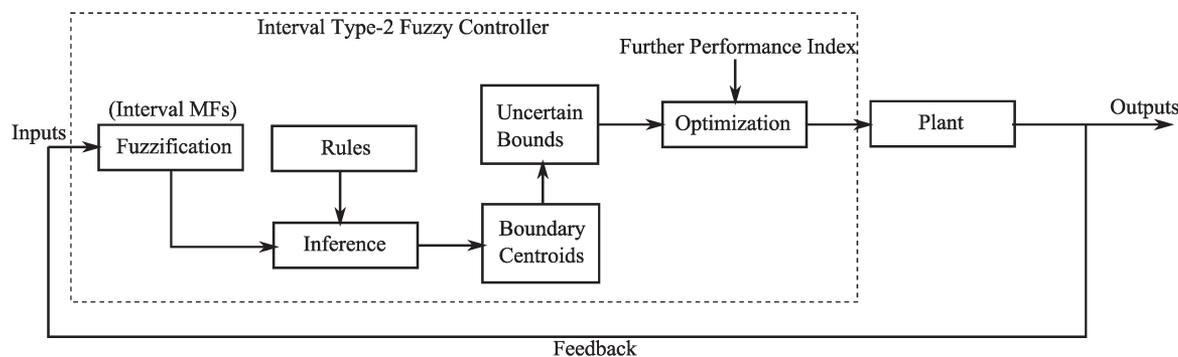


Fig. 3. Structure of the proposed IT2 T-S fuzzy controller.

principle of the IT2 T-S fuzzy system, a system model has the capability of switching among local linear models in an interval route constrained by certain bounds involving uncertainty and nonlinear dynamics. Equation (23) demonstrates that the center route of the interval terms is employed to build its global system model. It is crucial to identify a suitable structure within the principle to improve the performance of optimizing switching routes from bounded intervals.

### C. Proposed IT2 T-S Fuzzy Control System

A novel control structure is proposed in this section, which aims to integrate the IT2 reasoning findings with control performance optimization to rebuild the switching routes between local linear model controls. The structure is shown in Fig. 3.

By adapting the proposed structure, the crisp outputs of the IT2 T-S FLS can be recalculated as

$$\underline{u}_c^* = \min \{ u^{(O)}(t), u^{(M)}(t) \} \quad (24a)$$

$$\bar{u}_c^* = \max \{ u^{(O)}(t), u^{(M)}(t) \} \quad (24b)$$

$$\Gamma = f(\tilde{u}(t)), \quad \tilde{u}(t) \in (\underline{u}_c^* + \Delta u_c^*, \underline{u}_c^* + 2\Delta u_c^*, \dots, \bar{u}_c^*) \quad (24c)$$

$$\Delta u_c^* = \frac{\bar{u}_c^* - \underline{u}_c^*}{n} \quad (25)$$

where  $u^{(O)}(t)$  and  $u^{(M)}(t)$  can be calculated from (21a) and (21b),  $n$  denotes the resampling number,  $\Gamma$  denotes the further-optimization goal, and  $f$  is defined as a performance function of the system with variable  $\tilde{u}(t)$ . The control output  $\tilde{u}(t)$  can be solved from (24c) by off-the-shelf optimization algorithms.

With the preceding information, the systematic control procedure of the proposed framework is given here.

- 1) Determine all the state variables, their interval type-2 fuzzy membership functions (MFs), and fuzzy rules.
- 2) With the control plant and required control aims, design the optimization task, and choose the related proper optimization method.
- 3) Obtain the system inputs. The interval outputs are calculated with the interval type-2 fuzzy inference and the TR method by (13), (16), and (21a)–(22b).
- 4) Calculate the fuzzy control outputs by the further-optimization structure with (24a)–(25).

- 5) Perform the control outputs on the plant. The system inputs are updated, and the system performance in the further-optimization part is also recalculated.
- 6) Return to step 3 to do the next interval type-2 fuzzy reasoning. Recycle this process until the expected system performance is obtained.

In comparison with the conventional IT2 T-S FLS, the proposed structure built a more general framework to represent the defuzzifier processing. If an optimal goal of the proposed IT2 T-S FLS can be described by (23), the convergence of the optimization method is guaranteed, and the general method is shrunk to the same form as the conventional IT2 T-S FLS. However, under the proposed structure, the crisp output of the IT2 T-S FLS represents twofold information. One is the fuzzy rules extracted from expert knowledge or industrial experience. The other is the further optimal goal, which is required by practical issues or is impossible to be combined into the fuzzy rules. Optimization algorithms can be selected in terms of domain-dependent goals and practical requirements. For the purpose of evaluating the proposed method, a numerical example and a case study on a nonlinear half-vehicle active suspension system are implemented in Section V.

### IV. STABILITY ANALYSIS OF THE IT2 T-S FUZZY CONTROL SYSTEM

Stability is one of the most important issues in the analysis and design of control systems. Stability analysis of the fuzzy control system has been more difficult, because the system is essentially nonlinear. Receiving the existing stability analysis results of typical fuzzy control systems, type-1 T-S fuzzy control systems provided great development of systematic approaches to stability analysis in view of powerful conventional control theory and techniques [27]. However, there is only little attention to be given to the stability of type-2 fuzzy systems [28]–[33].

In this section, we analyze the stability of the proposed closed-loop IT2 T-S fuzzy control system, which is formed by the IT2 T-S fuzzy model in (13) and the proposed controller. To formulate the system in a closed format and without losing the generality, the control output can be rewritten as

$$u(t) \approx \sum_{i=1}^m (\alpha \underline{\omega}^i + (1 - \alpha) \bar{\omega}^i) K_i x, \quad \alpha \in [0, 1]. \quad (26)$$

Then, the closed-loop IT2 T-S fuzzy control system can be described as follows:

$$x(t+1) = \sum_{i=1}^m \sum_{j=1}^m G_{ij}(A_i + B_i K_j)x(t) \quad (27)$$

where  $G_{ij}$  denotes the fixed membership grade from the IT2 antecedents and T-S consequent and is described as

$$G_{ij} = [\alpha \underline{\omega}_i + (1 - \alpha) \bar{\omega}_i] \tilde{\omega}_j = \omega_i \tilde{\omega}_j \quad (28)$$

where  $\underline{\omega}_i$ ,  $\bar{\omega}_i$ , and  $\tilde{\omega}_j$  are defined in (11), (12), and (14), respectively.

For further stability analysis, (27) can be represented as a general uncertain system

$$\begin{aligned} x(t+1) &= G_0 x(t) + \sum_{i=1}^m \sum_{j=1}^m \omega_i \tilde{\omega}_j \Delta G_{ij} x(t) \\ &= G_0 x(t) + \sum_{i=1}^m \omega_i \tilde{\omega}_i \Delta G_{ii} x(t) \\ &\quad + \sum_{i < j}^m \omega_i \tilde{\omega}_j \Delta F_{ij} x(t) \\ &= \{G_0 + W \Delta(t) Z\} x(t). \end{aligned} \quad (29)$$

Here

$$\begin{aligned} G_0 &= \frac{1}{m} \sum_{i=1}^m (A_i + B_i K_i) \\ \Delta G_{ij} &= A_i + B_i K_j - G_0 \\ \Delta G_{ii} &= Q_{ii} \Phi_{ii} S_{ii}^T \\ \Delta F_{ij} &= \Delta G_{ij} + \Delta G_{ji} = Q_{ij} \Phi_{ij} S_{ij}^T, \quad i < j \end{aligned} \quad (31)$$

where  $Q$  and  $S$  are unitary matrices,  $W \in \mathbb{R}^{\nu \times \gamma}$ ,  $\Delta(t) \in \mathbb{R}^{\gamma \times \gamma}$ ,  $Z \in \mathbb{R}^{\gamma \times \nu}$ ,  $\gamma = [\nu \times m \times (m + 1)]/2$ , and matrices  $W$  and  $Z$  are given as follows:

$$\begin{aligned} W &= [\bar{Q}_1 \quad \bar{Q}_2 \quad \cdots \quad \bar{Q}_m] \\ Z &= [\bar{S}_1 \quad \bar{S}_2 \quad \cdots \quad \bar{S}_m]^T \\ \Delta(t) &= \text{block} - \text{diag} [\bar{\Phi}_1^e \quad \bar{\Phi}_2^e \quad \cdots \quad \bar{\Phi}_m^e] \end{aligned} \quad (32)$$

where

$$\begin{aligned} \bar{Q}_i &= [Q_{ii} \quad Q_{ii+1} \quad \cdots \quad Q_{ir}] \\ \bar{S}_i &= [S_{ii} \quad S_{ii+1} \quad \cdots \quad S_{ir}] \\ \bar{\Phi}_i^e &= \text{block} - \text{diag} [e_{ii} \Phi_{ii} \quad e_{ii+1} \Phi_{ii+1} \quad \cdots \quad e_{ir} \Phi_{ir}] \\ e_{ii} &= \omega_i \tilde{\omega}_j \in [\underline{\omega}_i \tilde{\omega}_j, \bar{\omega}_i \tilde{\omega}_j]. \end{aligned} \quad (33)$$

Based on (32), matrices  $M$  and  $N$  are defined as follows:

$$M = N = \text{block} - \text{diag} [\bar{\Phi}_1^d \quad \bar{\Phi}_2^d \quad \cdots \quad \bar{\Phi}_r^d] \quad (34)$$

where

$$\begin{aligned} \bar{\Phi}_i^d &= \text{block} - \text{diag} \left[ \frac{d_{ii}}{2} \Phi_{ii} \quad \frac{d_{ii+1}}{2} \Phi_{ii+1} \quad \cdots \quad \frac{d_{ir}}{2} \Phi_{ir} \right] \\ d_{ij} &= \max \omega_i \tilde{\omega}_j. \end{aligned} \quad (35)$$

Considering the IT2 membership grade of antecedents  $\omega_i \in [\underline{\omega}_i, \bar{\omega}_i]$ , if the interval bounds are fixed, then the stability of the IT2 T-S fuzzy system can be covered by the stability results of type-1 T-S fuzzy systems [34]. The stability analysis result is summarized in the following theorem:

*Theorem 1:* The equilibrium of an IT2 T-S fuzzy control system, as given in (27), formed by the TR method and control structure in Section III-C is quadratically stable if and only if one of the following conditions is satisfied:

- 1) There exists a positive definite matrix  $P$  such that

$$P(G_0 + WMZ) + (G_0 + WMZ)^T P + PWN N^T W^T P + Z^T Z < 0.$$

- 2) If defined as

$$H = \begin{bmatrix} G_0 + WMZ & -WNN^T W^T \\ Z^T Z & -(G_0 + WMZ) \end{bmatrix}$$

the condition is

$$\text{Re} \lambda_i(H) \neq 0, \quad i = 1, 2, \dots, 2 \times n.$$

- 3) There exists a positive definite matrix  $P$  such that

$$\begin{bmatrix} P(G_0 + WMZ) + (G_0 + WMZ)^T P & PWN & Z^T \\ N^T W^T P & -I & 0 \\ Z & 0 & -I \end{bmatrix} < 0.$$

*Remark 1:* Since the TR method is used to aggregate the IT2 centroids to bounds, the closed-loop system is reduced to type-1 T-S FLS, and its stability analysis is similar to the type-1 T-S FLS. It can also be deduced that, if all the subsystems related to the interval bounds are asymptotically stable, then the IT2 FLS is asymptotically stable. However, these stability conditions are only sufficient conditions with strong constraints.

*Remark 2:* Considering the proposed control structure in Fig. 3, algorithms can further be selected to indirectly tune  $G_{ij}$  by optimizing the uncertainty lower and upper bounds. Theorem 1 proves that, if one of the conditions in the theorem is satisfied, the optimization algorithms do not affect the closed-loop stability of the proposed IT2 T-S FLS.

## V. SIMULATION EXAMPLES

The proposed approach is implemented into a numerical example and a case study on a half-vehicle active suspension model in this section.

### A. Numerical Example

A numerical example is conducted to demonstrate the effect of the novel IT2 T-S FLS.

Consider a T-S fuzzy model with the following two rules:

- 1)  $R^1$ : If  $x$  is  $A_1$ , then  $y = 0.2x + 9$ .
- 2)  $R^2$ : If  $x$  is  $A_2$ , then  $y = 0.6x + 2$ .

This example has been used by Takagi and Sugeno [35] to illustrate the T-S model fuzzy reasoning results. Here, to demonstrate the reasoning process of the IT2 FLS, the proposed

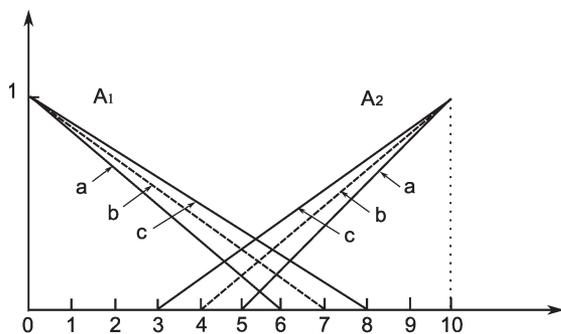


Fig. 4. Interval membership functions denote the (a) lower MFs, (b) crisp MFs, and (c) upper MFs.

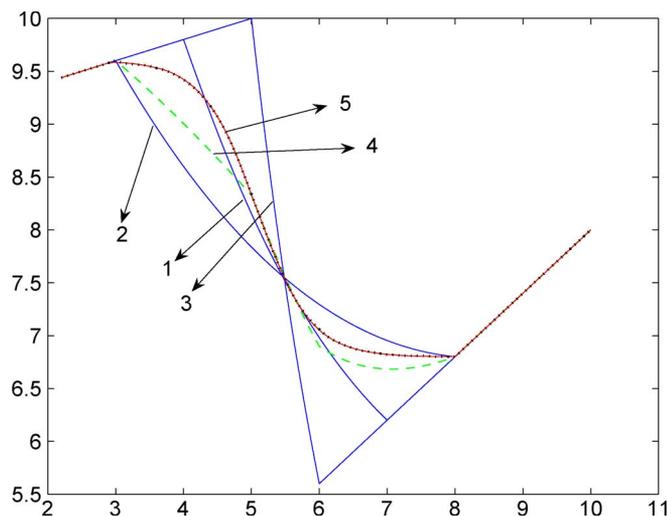


Fig. 5. Interval fuzzy reasoning results (1: type-1 reasoning middle route; 2: type-1 reasoning lower route; 3: type-1 reasoning upper route; 4: K–M IT2 reasoning route; 5: proposed IT2 reasoning route).

interval membership functions are implemented in the same example. The membership functions of  $A_1$  and  $A_2$  are shown in Fig. 4. The figure includes not only the interval membership functions shown as  $a$  and  $c$  but the crisp membership functions denoted as  $b$  from [35] as well.

Fig. 5 showed the simulation results generated by a type-1 T–S fuzzy system, an IT2 T–S fuzzy system, and the proposed IT2 T–S fuzzy system with further optimization. First, translated route 1 was obtained by using type-1 membership functions  $b$  in Fig. 4 and the type-1 fuzzy reasoning method in [35]; routes 2 and 3 were produced by employing boundary membership functions  $a$  and  $c$  in Fig. 4 and the same type-1 reasoning method. It illustrated that the sliding route from one line to the other was nonlinear and with a boundary field between the lower and upper routes. It implied that type-1 fuzzy reasoning can model the nonlinear switch routes between two linear surfaces but cannot deal with uncertainties in the switching routes.

By using the IT2 membership functions and K–M algorithm, switching route 4 was obtained. It is different from all the type-1 fuzzy reasoning results and piecewise near route 2 (i.e., the input value in [3, 4] and [7, 8]) and route 1 (i.e., the input value in [5, 6]). Route 4 also shows that IT2 T–S FLS has the inherent ability to build more complex switch routes than type-1 FLS. Route 5 was produced by employing the proposed IT2 T–S FLS

TABLE I  
PARAMETERS OF HALF-VEHICLE ACTIVE SUSPENSION

$m_b$ (Kg)	$I_b$ (Kg.m <sup>2</sup> )	$m_{wf}$ (Kg)	$m_{wr}$ (Kg)	$d_f$ (m)	$d_r$ (m)
1794.4	3443.05	187.15	440.04	1.15	1.60
$c_{f1}$ (m)	$c_{f2}$ (m)	$k_{0f}$ (N)	$k_{1f}$ (N/m)	$k_{2f}$ (N/m <sup>2</sup> )	$k_{3f}$ (N/m <sup>3</sup> )
1190	426	-136	60824	-10865	104
$c_{1r}$ (m)	$c_{2r}$ (m)	$k_{0r}$ (N)	$k_{1r}$ (N/m)	$k_{2r}$ (N/m <sup>2</sup> )	$k_{3r}$ (N/m <sup>3</sup> )
1000	215	-146	18615	-3665	384

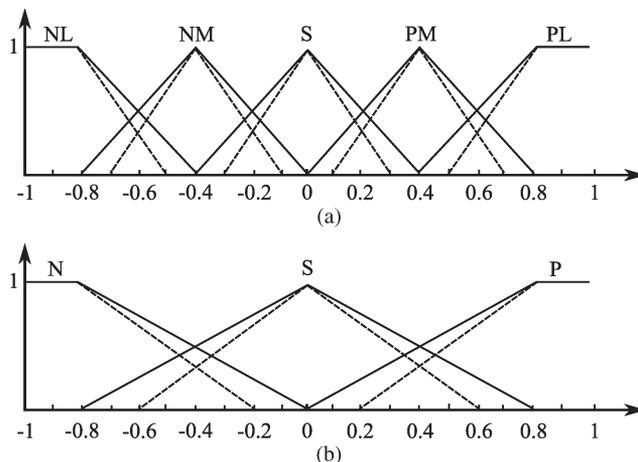


Fig. 6. IT2 fuzzy membership functions of the four FLC input variables. (a) Membership functions of the vehicle body velocities (i.e.,  $\dot{z}_{bf}$  and  $\dot{z}_{br}$ ). (b) Membership functions of the displacements of the vehicle body (i.e.,  $z_{bf}$  and  $z_{br}$ ).

with the optimal goal in (36). Switching route 5 was optimally generated among the linear control surfaces to track the sigmoid function

$$f(x) = \frac{2.8}{1 + e^{2.5(x-5.08)}} + 6.8. \quad (36)$$

The reasoning results demonstrated that the proposed structure can make use of interval type-2 fuzzy reasoning ability and the proposed structure to rebuild the switching lines between linear surfaces and that it has the potential ability to deal with the high nonlinearity and uncertainty.

### B. Half-Vehicle Active Suspension System

The proposed method in Section III was implemented into a half-vehicle active suspension system, as shown in Fig. 1, whose mathematical model is given in Section II. The parameters of the model are provided in Table I. The vehicle body velocities (i.e.,  $\dot{z}_{bf}$  and  $\dot{z}_{br}$ ) and displacements (i.e.,  $z_{bf}$  and  $z_{br}$ ) are chosen as inputs, and the actuator forces (i.e.,  $U_f$  and  $U_r$ ) are chosen as outputs. The interval membership functions of the inputs are provided in Fig. 6. The consequents are linear control outputs, as given in

$$U = \begin{bmatrix} U_f \\ U_r \end{bmatrix} = -KX \quad (37)$$

TABLE II  
RULES OF FUZZY CONTROLLER

$\dot{z}_{bf}$	$\dot{z}_{br}$	$z_{bf}$	$z_{br}$	$U$	$\dot{z}_{bf}$	$\dot{z}_{br}$	$z_{bf}$	$z_{br}$	$U$
S or NM	S or NM	S	S	$-K_0X$	NL	NL	S	S	$-K_4X$
PM or PL	S or NM	S	S	$-K_1X$	S or NM	S or NM	P or N	P or N	$-K_3X$
NL	S or NM	S	S	$-K_3X$	PM or PL	S or NM	P or N	P or N	$-K_0X$
S or NM	PM or PL	S	S	$-K_1X$	NL	S or NM	P or N	P or N	$-K_3X$
PM or PL	PM or PL	S	S	$-K_3X$	PM or PL	S or NM	P or N	P or N	$-K_0X$
NL	PM or PL	S	S	$-K_3X$	PM or PL	PM or PL	P or N	P or N	$-K_1X$
S or NM	NL	S	S	$-K_3X$	NL	PM or PL	P or N	P or N	$-K_3X$
PM or PL	NL	S	S	$-K_0X$	S or NM	NL	P or N	P or N	$-K_2X$
PM or PL	NL	P or N	P or N	$-K_1X$	NL	NL	P or N	P or N	$-K_2X$

where

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \quad (38)$$

$$X = \begin{bmatrix} \dot{Z}_{bf} \\ \dot{Z}_{br} \\ Z_{bf} \\ Z_{br} \end{bmatrix} \quad (39)$$

The vehicle speed is 20 m/s.

Based on the linear model of the half-vehicle suspension system and linear quadratic (LQ) control strategy, the control gains  $K$  can be solved [24]. There are a total of 18 fuzzy rules for the half-vehicle suspension control system, as shown in Table II. The local linear controller gains are given as follows:

$$K_0 = \begin{bmatrix} -28.9 & -4095.1 & -69.3 & -66341.6 \\ -3568.9 & -30.58 & -19019.6 & -73.34 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -28.9 & -4095.1 & -69.3 & -76365.6 \\ -3568.9 & -30.58 & -26874.6 & -73.34 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -28.9 & -4095.1 & -69.3 & -70365.6 \\ -3568.9 & -30.58 & -22874.66 & -73.34 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -28.9 & -4095.1 & -69.3 & -60541.6 \\ -3568.9 & -30.58 & -15474.66 & -73.34 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} -28.9 & -4095.1 & -69.3 & -57541.6 \\ -3568.9 & -30.58 & -13474.66 & -73.34 \end{bmatrix}$$

The fuzzy rules and the local linear controller are mainly designed to reduce the body accelerations with the aim to improve the riding comfort. Herein, the proposed new control structure with a cost function in (40) is employed to save the actuator's energy

$$\Gamma(U_f, U_r) = \min \sqrt{(q_1 \ddot{z}_{bf}^2 + q_2 \ddot{z}_{br}^2 + q_3 U_f^2 + q_4 U_r^2)} \quad (40)$$

where, to prioritize the parameters,  $q_1$  is set as 1,  $q_2$  is set as 1,  $q_3$  is set as 10, and  $q_4$  is set as 10. Due to the real-time requirement and its nonlinearity, a particle swarm optimization algorithm proposed in [36] was integrated into the structure. In addition, the resample number is set as 20 by default, the inertia weight is 0.5, and the acceleration coefficients for local optimization and global optimization are 2.

For evaluation purposes, an LQ controller [24] and an IT2 fuzzy controller with Wu–Mendel uncertain bounds [3] are also designed to compare with the proposed approach.

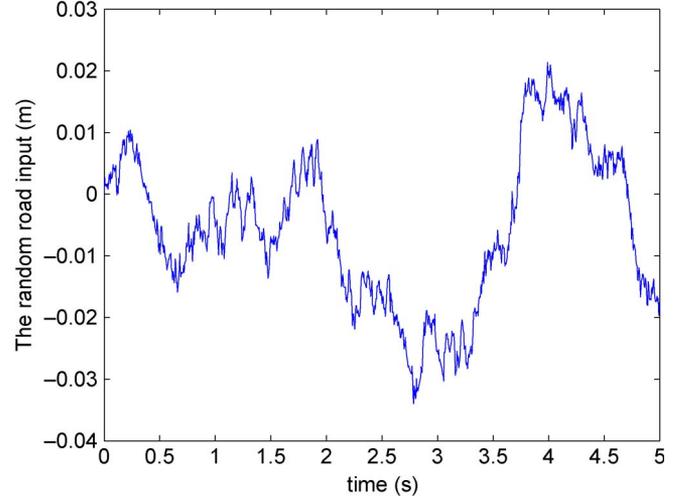


Fig. 7. Random road inputs.

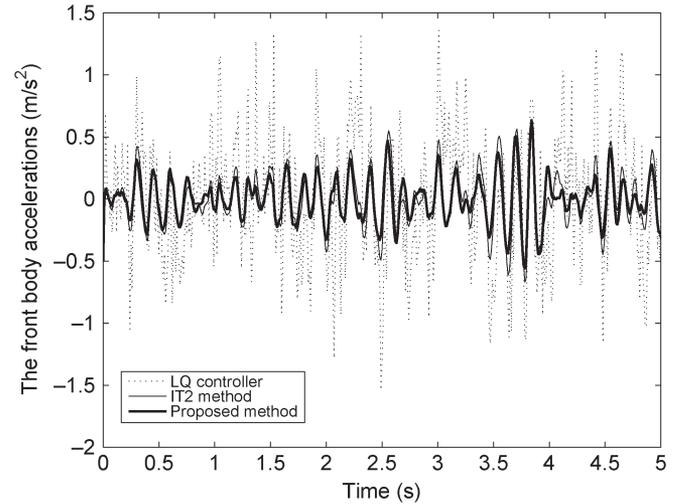


Fig. 8. Front body accelerations with random road inputs.

The class-C road surface, which is one kind of poor road surface, is used as a random road input, where the road roughness is  $2.56 \times 10^{-4} \text{ m}^3/\text{cycle}$  according to the ISO (1982) classification using the power spectral density, as shown in Fig. 7.

The simulation results are shown in Figs. 8–11. Regarding the accelerations of the front and rear bodies, the proposed method has achieved better performance on riding comfort than the other two methods. On the other hand, the linear controller has not satisfied the criteria related to the nonlinearity of the

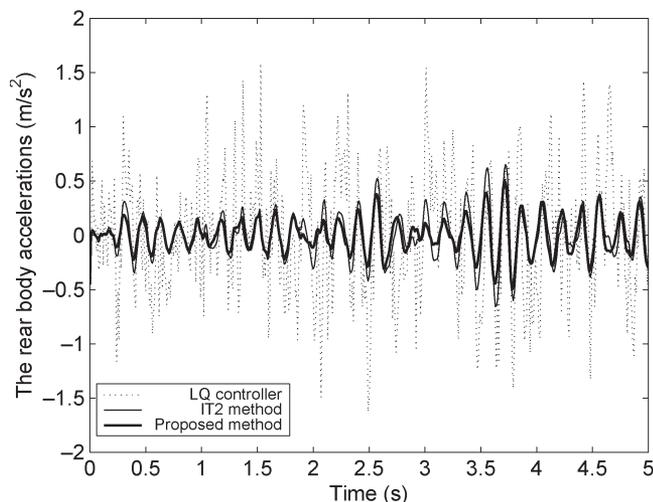


Fig. 9. Rear body accelerations with random road inputs.

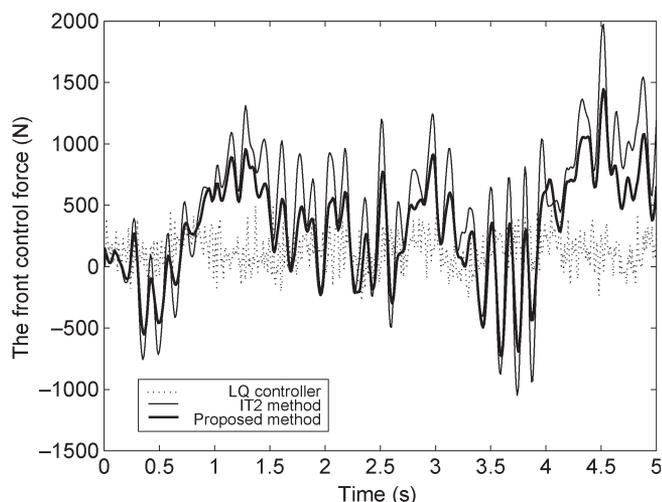


Fig. 10. Front control force with random road inputs.

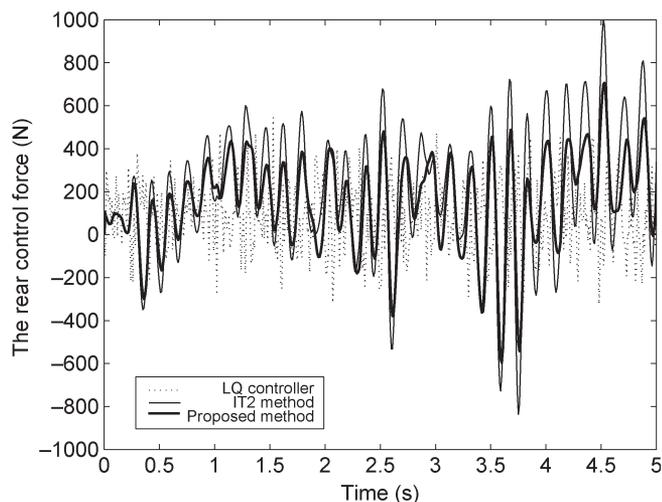


Fig. 11. Rear control force with random road inputs.

suspension system. Figs. 10 and 11 showed the comparison of the control force. It is evident that the proposed method requires lower force than the conventional IT2 method but higher force than the LQ controller.

TABLE III  
RMS VALUE COMPARISON OF BODY ACCELERATIONS AND CONTROL FORCES

	FVA ( $m/s^2$ )	RVA ( $m/s^2$ )
LQ Controller	0.5067	0.5652
IT2 Method	0.2195	0.2122
Proposed Method	0.1811	0.1608
	Front control forces (N)	Rear control forces (N)
LQ Controller	191.85	176.09
IT2 Method	715.23	350.42
Proposed Method	519.93	261.14

<sup>a</sup>FVA: Front Vehicle Accelerations, RVA: Rear Vehicle Accelerations;

TABLE IV  
CREST FACTOR COMPARISON OF BODY ACCELERATIONS

Random Road Input	Front Vehicle	Rear Vehicle
LQ Controller	3.1422	3.3526
IT2 method	2.8366	2.2031
Proposed method	2.4235	2.1125

TABLE V  
RIDE INDEX COMPARISON OF VEHICLE BODY

Random Road Input	Front Vehicle	Rear Vehicle
LQ Controller	0.3621	0.3765
IT2 Method	0.2874	0.2930
Proposed Method	0.2655	0.2512

From the statistic evaluation point, two kinds of performance criteria are used to compare the vehicle active suspension control performance. One is the RMS value, which presents the vehicle ride comfort and handling performance from the time domain [1]. Another is the ride index of body vibration, which focuses on the ride comfort from frequency-weighted vibrating accelerations [37].

The comparison of RMS values for vertical accelerations and control forces is shown in Table III.

Based on the ISO criteria of vehicle ride comfort evaluation, the crest factors of body vertical and rotational vibrations are determined in Table IV. The ride index calculation method in [38] is used to compare the vehicle ride comfort with LQ controller, IT2 fuzzy control system, and proposed fuzzy control system. Table V shows the comparison of the ride index.

Regarding the RMS accelerations and ride index of the front and rear bodies, the proposed method has achieved better performance on ride comfort than the other two methods. Simultaneously, simulation results showed that the proposed method needs higher control force than the LQ control system.

## VI. CONCLUDING REMARKS

This paper has presented a novel IT2 T–S fuzzy control system for active suspensions to resolve their nonlinear dynamics, more deeply understand the uncertainty embedded in fuzzy linguistic rules and reasoning, and meet real-time requirements as a whole. The proposed approach has integrated the IT2 membership functions, T–S fuzzy model, Wu–Mendel uncertain bounds, and further-optimization algorithms into one control framework. It has been implemented into case studies and been compared with the existing IT2 fuzzy control system. It is evident that the proposed approach outperforms the LQ controller and the IT2 fuzzy controller. Simulation results have shown that the proposed IT2 T–S fuzzy control system cannot only effectively handle the system uncertainty and improve control performance but can also save actuator energy with an added optimization module. The priority of future work is given to the real-time applications on an electric vehicle suspension system. In addition, in-depth interpretation on uncertainty bounds will be investigated based on interval reasoning. The stability conditions of the proposed closed-loop systems will further be relaxed. Future work is also aimed at a bit more ambitious target of developing a fuzzy inference engine, which has the capability of conducting reasoning in terms of qualitative and probabilistic information [39], [40].

## REFERENCES

- [1] D. Hrovat, "Survey of advanced suspension developments and related optimal control applications," *Automatica*, vol. 33, no. 10, pp. 1781–1817, Oct. 1997.
- [2] M. Nagai, "Recent researches on active suspensions for ground vehicles," *Jpn. Soc. Mech. Eng.*, vol. 36, no. 2, pp. 161–170, 1993.
- [3] J. Cao, H. Liu, P. Li, and D. Brown, "Adaptive fuzzy logic controller for vehicle active suspensions with interval type-2 fuzzy membership functions," in *Proc. IEEE World Congr. Comput. Intell.*, 2008, pp. 83–89.
- [4] C. Ting, T. Li, and F. Kung, "Design of fuzzy controller for active suspension system," *Mechatronics*, vol. 5, no. 4, pp. 365–383, Jun. 1995.
- [5] S. Huang and W. Lin, "Adaptive fuzzy controller with sliding surface for vehicle suspension control," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 550–559, Aug. 2003.
- [6] S. Kumarawadu and T. Lee, "Neuroadaptive combined lateral and longitudinal control of highway vehicles using RBF networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 7, no. 4, pp. 500–512, Dec. 2006.
- [7] S. Huang and C. Lin, "Application of a fuzzy enhance adaptive control on active suspension system," *Int. J. Comput. Appl. Technol.*, vol. 20, no. 4, pp. 152–160, May 2004.
- [8] I. Kucukdemiral, S. Engin, V. Omurlu, and G. Cansever, "A robust single input adaptive sliding mode fuzzy logic controller for automotive active suspension system," in *Fuzzy Systems and Knowledge Discovery*. New York: Springer-Verlag, 2005, pp. 981–986.
- [9] S. Wu, C. Wu, and T. Lee, "Neural-network-based optimal fuzzy control design for half-car active suspension systems," in *Proc. IEEE Intell. Vehicles Symp.*, 2005, pp. 376–381.
- [10] R. Saecks, C. Cox, J. Neidhoefer, P. Mays, and J. Murray, "Adaptive control of a hybrid electric vehicle," *IEEE Trans. Intell. Transp. Syst.*, vol. 3, no. 4, pp. 213–234, Dec. 2002.
- [11] R. Lian, B. Lin, and W. Sie, "Self-organizing fuzzy control of active suspension systems," *Int. J. Syst. Sci.*, vol. 36, no. 3, pp. 119–135, Feb. 2005.
- [12] H. Hagrass, "A hierarchical type-2 fuzzy logic control architecture for autonomous mobile robots," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 4, pp. 524–539, Aug. 2004.
- [13] R. Sepúlveda, O. Castillo, P. Melin, A. Rodríguez-Díaz, and O. Montiel, "Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic," *Inf. Sci.*, vol. 177, no. 10, pp. 2023–2048, May 2007.
- [14] O. Castillo and P. Melin, *Type-2 Fuzzy Logic: Theory and Applications*, 1st ed. Heidelberg, Germany: Springer-Verlag, Jan. 2008.
- [15] L. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning. Part 1," *Inf. Sci.*, vol. 8, no. 3, pp. 199–249, 1975.
- [16] J. Mendel, "Advances in type-2 fuzzy sets and systems," *Inf. Sci.*, vol. 177, no. 1, pp. 84–110, Jan. 2007.
- [17] Q. Liang and J. Mendel, "Interval type-2 fuzzy logic systems: Theory and design," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 535–550, Oct. 2000.
- [18] H. Wu and J. Mendel, "Uncertainty bounds and their use in the design of interval type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 5, pp. 622–639, Oct. 2002.
- [19] J. Mendel, H. Hagrass, and R. John, "Standard Background Material About Interval Type-2 Fuzzy Logic Systems That Can Be Used by All Authors," [Online]. Available: [http://iee-cis.org/\\_files/standards.t2.win.pdf](http://iee-cis.org/_files/standards.t2.win.pdf)
- [20] N. Karnik and J. Liang, "Type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 6, pp. 643–658, Dec. 1999.
- [21] N. Karnik and J. Mendel, "Centroid of a type-2 fuzzy set," *Inf. Sci.*, vol. 132, no. 1–4, pp. 195–220, Feb. 2001.
- [22] S. Coupland and R. John, "A fast geometric method for defuzzification of type-2 fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 4, pp. 929–941, Aug. 2008.
- [23] M. Nie and W. Tan, "Towards an efficient type-reduction method for interval type-2 fuzzy logic systems," in *Proc. IEEE Int. Conf. Fuzzy Syst.*, 2008, pp. 1425–1432.
- [24] J. Cao, H. Liu, P. Li, and D. Brown, "Study on active suspension control system based on an improved half-vehicle model," *Int. J. Autom. Comput.*, vol. 4, no. 3, pp. 236–242, 2007.
- [25] C. Kim and P. Ro, "A sliding mode controller for vehicle active suspension systems with non-linearities," *Proc. IMechE D, J. Automobile Eng.*, vol. 212, no. 2, pp. 79–92, 1998.
- [26] Q. Liang and J. Mendel, "Equalization of nonlinear time-varying channels using type-2 fuzzy adaptive filters," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 551–563, Oct. 2000.
- [27] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 676–697, Oct. 2006.
- [28] N. Cárdenas, S. Cárdenas, L. Aguilar, and O. Castillo, "Lyapunov stability on type-2 fuzzy logic control," in *Proc. IEEE-CIS Int. Sem. Comput. Intell.*, México City, DF, México, 2005, pp. 1–10.
- [29] O. Castillo, N. Cárdenas, D. Rico, and L. Aguilar, "Intelligent control of dynamic systems using type-2 fuzzy logic and stability issues," *Int. Math. Forum*, vol. 1, no. 28, pp. 1371–1382, 2006.
- [30] O. Castillo, L. Aguilar, N. Cárdenas, and S. Cárdenas, "Systematic design of a stable type-2 fuzzy logic controller," *Appl. Soft Comput. J.*, vol. 8, no. 3, pp. 1274–1279, Jun. 2008.
- [31] H. K. Lam and L. D. Seneviratne, "Stability analysis of interval type-2 fuzzy-model-based control systems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, no. 3, pp. 617–628, Jun. 2008.
- [32] B. M. Begian, W. W. Melek, and J. Mendel, "Stability analysis of type-2 fuzzy systems," in *Proc. IEEE World Congr. Comput. Intell.*, 2008, pp. 947–953.
- [33] J. Morales, O. Castillo, and J. Soria, "Stability on type-1 and type-2 fuzzy logic systems," in *Soft Computing for Hybrid Intelligent Systems*. New York: Springer-Verlag, 2008, pp. 29–51.
- [34] K. Tanaka, T. Ikeda, and H. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability,  $H_\infty$  control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 1–13, Feb. 1996.
- [35] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, no. 1, pp. 116–132, Feb. 1985.
- [36] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE Int. Conf. Neural Netw.*, 1995, vol. 4, pp. 1942–1948.
- [37] Mechanical Vibrations and Shock—Evaluation of Human Exposure to Whole-Body Vibration. Part 1: General Requirements, ISO2631-1, 1997.
- [38] M. Montazeri-Gh and M. Soleymani, "Genetic optimization of a fuzzy active suspension system based on human sensitivity to the transmitted vibrations," *Proc. Inst. Mech. Eng. D, J. Automobile Eng.*, vol. 222, no. 10, pp. 1769–1780, 2008.
- [39] H. Liu, D. Brown, and G. Coghill, "Fuzzy qualitative robot kinematics," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 808–822, Jun. 2008.
- [40] Z. Liu and H. Li, "A probabilistic fuzzy logic system for modeling and control," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 6, pp. 848–859, Dec. 2005.



**Jiangtao Cao** (M'07) received the B.Sc. and M.Sc. degrees from Liaoning Shihua University, Fushun, China, in 2000 and 2003, respectively, and the Ph.D. degree in intelligent control from the University of Portsmouth, Portsmouth, U.K., in 2009.

He is currently a Professor with the School of Information and Control Engineering, Liaoning Shihua University. He has published more than 20 research papers. His research interests include active suspension control, intelligent robotics, qualitative reasoning, fuzzy control theory, and applications.



**Ping Li** (M'00–SM'08) received the B.Sc. and M.Sc. degrees from Northwestern Polytechnical University, Xi'an, China, in 1984 and 1987, respectively, and the Ph.D. degree in automatic control from Zhejiang University, Hangzhou, China, in 1995.

He is currently a Full Professor with Liaoning Shihua University, Fushun, China, an Adjunct Professor and Ph.D. Advisor with Northwestern Polytechnical University, and a Visiting Professor with the University of Portsmouth, Portsmouth, U.K. He has published more than 100 papers and successfully

completed more than 20 research projects supported by national and ministry funds. His research interests include process control and automation, particularly advanced control and optimization of chemical industry process control systems.

Prof. Li has received more than ten awards from the nation and ministries.



**Honghai Liu** (M'02–SM'06) received the Ph.D. degree in robotics from King's College London, London, U.K., in 2003.

In September 2005, he joined the University of Portsmouth, Portsmouth, U.K. He previously held research appointments with the University of London and the University of Aberdeen, Aberdeen, U.K., and project leader appointments in the industrial control and system integration industry. He has published more than 150 research papers. His research interests are computational intelligence methods and applications, with a focus on those approaches that could make contributions to the intelligent connection of perception to action.

Dr. Liu has received three Best Paper Awards.