Modeling of Plug-in Hybrid Electric Vehicle Charging Demand in Probabilistic Power Flow Calculations

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***Abstract*—Millions of electric vehicles (EVs), especially plug-in hybrid EVs (PHEVs), will be integrated into the power grid in the near future. Due to their large quantity and complex charging behavior, the impact of substantial PHEVs charging on the power grid needs to be investigated. Since the charging behavior of PHEVs in a certain regional transmission network or a local distribution network is determined by different uncertain factors, their overall charging demand tends to be uncertain and in this situation probabilistic power flow (PPF) can be applied to analyze the impact of PHEVs charging on the power grid. However, currently there is no suitable model of the overall charging demand of PHEVs available for PPF calculations. In this paper, a methodology of modeling the overall charging demand of PHEVs is proposed. The proposed methodology establishes a single PHEV charging demand model, and then employs queuing theory to describe the behavior of multiple PHEVs. Moreover, two applications are given, i.e., modeling the overall charging demand of PHEVs at an EV charging station and in a local residential community, respectively. Comparison between PPF calculations and Monte Carlo simulation are made on a modified IEEE 30-bus system integrated with the two demand models proposed.**

***Index Terms*—Charging demand model, Cornish-Fisher expansion, cumulant method, plug-in hybrid electric vehicle, point estimate method, probabilistic power flow.**

I. Introduction

As environmental pressure and energy depletion are increasingly severe, more and more attention has been paid to electric vehicles (EVs) because of their high energy efficiency and low off-gas emission compared to conventional internal combustion engine based vehicles [1]. Currently, due to technical limitations of power batteries, the EVs of the greatest potential are probably plug-in hybrid EVs (PHEVs), which compromise between cost and performance [2]. Both main industrial countries and large developing countries have made incentive policies for the popularization of PHEVs and leading manufactures are committed to PHEV technique development [3].

A PHEV is typically equipped with a drive train that at least contains an internal combustion engine, an electrical motor, a battery storage system and a means of recharging the battery system from an external source of electricity [2]. Its battery capacity is usually several kWh or more to power the vehicle in all electric drive mode for several tens of miles [4] and its internal combustion engine can be engaged to extend its drive range when the battery's charge is not sufficient [5].

In recent years, PHEVs are rapidly being accepted by more and more consumers. As the number of PHEVs increases rapidly, their charging demand will become an important part of the overall load in the power system. For example, the estimated number of PHEVs in America could reach over 1 million during 2015–2017, most of which are operated in the territory of main load centers, e.g., metropolitans; their overall charging demand could reach tens or even hundreds of MW in extreme situations [6]. Therefore, it is necessary to analyze the impact of substantial PHEVs charging on the power grid.

Related research on the impact of EV charging on the power grid started from 1980s. It was discovered that the charging demand is likely to coincide with the overall peak load [7] and it is necessary to manage the charging demand when the penetration of EVs increases, otherwise the overall peak load could increase significantly [8], [9]. Therefore, the concept of smart charging was proposed, which is aimed at optimizing the charging process of PHEVs [10]–[15]. A control strategy was proposed in [10] to optimize the energy consumption stemming from PHEV charging in a residential use case; another two strategies were presented in [11] to optimize charging time and energy flows of a PHEV, considering forecasted electricity price and system auxiliary service. In [12] and [13], the possible benefits of PHEVs as a certain type of auxiliary service were discussed in details and some conceptual framework for its implementation was presented in [14] and [15].

Although smart charging demonstrates a good application potential in the future smart grid, the consumers in reality may prefer to charge their PHEVs as fast as possible, so that smart charging control does not interfere with their daily drive profile [12], [13]. On the other hand, rapid charging techniques are also developing fast [16], which could attract more PHEV consumers. Moreover, the physical implementation and integration of smart charging is still to be done in a system wide scale in future. Therefore, currently it is still necessary to evaluate the impact of uncontrolled charging of PHEVs.

The charging behavior of PHEVs is affected by different factors, such as the number of PHEVs being charged, their charging voltage and current levels, power battery start/end status and capacity, and charging time duration etc. All these factors tend to be uncertain if all the PHEVs at an EV charging station or in a residential community are considered, so from an overall point of view their overall charging demand is uncertain as well.

Probabilistic power flow (PPF) is an essential tool for power system analysis [17], which takes uncertainty into consideration, thus it can be applied to analyze the influence of substantial PHEVs charging on the power grid. There has been

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some literature on the modeling of charging demand of PHEVs and analysis on its impact: A specified operation mode for charging PHEVs at home during day and night periods was assumed in [18], with given charging periods, charging level, battery start/end status and capacity, then a coordinated strategy of PHEV charging power was proposed to improve the voltage profile in the residential distribution grid. But this strategy was based on deterministic power flow analysis, so it cannot take uncertainty into consideration. Queuing theory [19] was first introduced in [20] to model the instantaneous demand of an EV charging station. This model was extended to describe the total charging and discharging power of PHEVs in a certain region in [21] for probabilistic constrained power flow calculations. However, only one type of PHEVs was considered in [20], [21], while other factors that affect the charging behavior of PHEVs, e.g., differences in battery capacity and charging level, were neglected. An analytic approach of modeling the daily recharge energy of EVs in a certain region was presented in [22]. Such an approach considered the types of PHEVs, the start/end status of charging power batteries and the charging periods as random distributions, and then obtains the inverted load duration curve of charging PHEVs as its random distribution for probabilistic production costing from the perspective of adequacy reliability. However, the drawback of the approach in [22] is the charging demand of PHEVs in the region of interest was simply added together, regardless of the influence between PHEVs themselves.

Besides their drawbacks in the consideration of uncertain factors, the models proposed in [19]–[22] generally expressed the overall charging demand of PHEVs as a nonlinear function of several random variates, which leads to great difficulty in calculate its numeric characteristics. However, if the overall charging demand needs to be considered in PPF calculations, its numeric characteristics become essential [23], [24]. Therefore, in order to overcome such a difficulty and at the same time to take more uncertain factors affecting the PHEV charging into consideration, a methodology of modeling the overall charging demand of PHEVs is proposed in this paper.

Combining the analytic approach in [22] and queuing theory [19], the proposed methodology employs random simulation and statistical analysis to fit a suitable distribution for the overall charging demand of PHEVs. Two applications of the proposed methodology are given, i.e., modeling the overall charging demand of PHEVs at an EV charging station and in a residential community by the proposed methodology respectively. In case studies, a comparison between PPF calculations and Monte Carlo simulation (MCS) is made in a modified IEEE 30-bus system, in order to validate the effectiveness of the proposed modeling methodology.

II. Methodology for Modeling of the Overall Charging Demand of PHEVs

A. Single PHEV Recharge Energy

In this section, the energy consumption for charging a PHEV is developed based on the probabilistic model proposed in [22], so that the main factors that determine the PHEV charging behavior are taken into consideration, such as its battery capacity, operating status, and daily driving range and so on.

Firstly, in order to describe the operating status of a PHEV, a key parameter k_{EV} is defined as

$$k_{\mathrm{EV}} = \frac{E_{\mathrm{Bat}}}{E_{\mathrm{Eng}} + E_{\mathrm{Bat}}} \tag{1}$$

where E_{Eng} is the total energy over a certain time period input into the vehicle engine and electric drive controller and E_{Bat} is the energy delivered at the same time by the onboard battery to the electric drive controller.

Obviously, k_{EV} represents the fraction of the total energy input supplied by the battery. For a charge-sustaining EV, since no energy is drawn from the battery, $k_{\mathrm{EV}} = 0$; For a zero-emission electric vehicle (ZEV), which is purely driven by the battery, $k_{\mathrm{EV}} = 1$; For a PHEV, the actual value of k_{EV} is somewhere between 0 and 1.

Secondly, another parameter of a PHEV is its total battery capacity C_{Bat}. The control strategy of a PHEV is assumed to adjust k_{EV}, i.e., its operating status, according to its C_{Bat} [25], so it can be assumed that k_{EV} and C_{Bat} are correlated and modeled as a bivariate normal distribution in this paper

$$\begin{bmatrix} k_{\mathrm{EV}} \\ C_{\mathrm{Bat}} \end{bmatrix} = \begin{bmatrix} \mu_{k_{\mathrm{EV}}} \\ \mu_{C_{\mathrm{Bat}}} \end{bmatrix} + \mathbf{L} \cdot \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \tag{2}$$

where $\mu_{k_{\mathrm{EV}}}$ and $\mu_{C_{\mathrm{Bat}}}$ are the means of k_{EV} and C_{Bat} respectively; $\boldsymbol{L}$ is the Cholesky decomposition of their covariance matrix $\sum$, i.e., $\sum = \boldsymbol{L} \cdot \boldsymbol{L}^{\mathrm{T}}$; N_1 and N_2 are two independent standard normal variates.

Thirdly, the performance of a PHEV can be assessed by its energy consumption per mile driven, denoted as E_m, which is approximately expressed as a function of k_{EV}

$$E_m = A_E \cdot (k_{\mathrm{EV}})^{B_E} \tag{3}$$

where the constant coefficients A_E and B_E are dependent on the PHEV type.

Fourthly, according to the PHEV driving pattern statistics in [26], the daily driven miles of a PHEV, denoted as M_d, tends to follow the lognormal distribution

$$M_d = e^{(\mu_m + \sigma_m \cdot N)} \tag{4}$$

where N is a standard normal variate. The lognormal distribution parameters μ_m and σ_m are calculated from the mean and standard variation of M_d, denoted as μ_{M_d} and σ_{M_d} respectively

$$\begin{cases} \mu_m = \ln\left(\dfrac{\mu_{M_d}^2}{\sqrt{\mu_{M_d}^2 + \sigma_{M_d}^2}}\right) \\ \sigma_m = \sqrt{\ln\left(1 + \dfrac{\sigma_{M_d}^2}{\mu_{M_d}^2}\right)} \end{cases} \tag{5}$$

Finally, after defining its energy consumption per mile E_m and daily driven range M_d, the daily recharge energy of a PHEV, denoted as D_E, is defined as follows:

$$D_E = \begin{cases} C_{\mathrm{Bat}} & M_d \geq M_E \\ M_d \cdot E_m & M_d < M_E \end{cases} \tag{6}$$

where constant M_E is the maximum driving distance of a PHEV in all electric driven mode, i.e.,

$$M_E = \frac{C_{\text{Bat}}}{E_m} = \frac{C_{\text{Bat}}}{\left(A_E \cdot k_{\text{EV}}^{B_E}\right)}. \tag{7}$$

B. Multiple PHEV Charging Demand

For multiple PHEVs plugged at load buses, queuing theory [19] can be employed to describe their overall charging process. For different situations of PHEV charging, different queuing models can be selected. In this paper, two typical scenarios of PHEV charging are presented, i.e., charging PHEVs at an EV charging station and in a local residential community.

PHEVs at an EV charging station can be considered as queuing customers to be served in an $M/M/c$ queue, where the first M denotes the inter-arrival time of customers following the exponential distribution with mean $T_\lambda (T_\lambda > 0)$, the second M denotes the service time of a customer for a customer following the exponential distribution with mean $T_\mu (T_\mu > 0)$ and c denotes the maximum customer number being served at the same time.

It should be explained that the exponential distribution is employed here because the PHEVs are assumed to be independent in their arrival and charging duration, i.e., both their arrival and charging are the Poisson process. On the other hand, the number of customers waiting at the EV charging station is assumed to be infinite for simplicity.

In terms of queuing theory, the number of PHEVs being charged at the same time in an $M/M/c$ queue, denoted as n, follows a discrete distribution as follows:

$$p_n = \begin{cases} \left(\sum_{i=0}^{c-1} \frac{(c\rho)^i}{i!} + \frac{(c\rho)^c}{c!} \cdot \frac{1}{1-\rho}\right)^{-1} & n = 0 \\ \frac{(c\rho)^n}{n!} \cdot p_0 & n = 1, 2, \ldots, c. \end{cases} \tag{8}$$

where ρ is the occupation rate per server defined as follows:

$$\rho = \frac{T_\mu}{cT_\lambda}. \tag{9}$$

It should be noted that it is generally required that the occupation rate $\rho < 1$, so that the length of the PEHV queue does not explode.

As for the PHEVs in a residential community, the $M/M/c/k/N_{\max}$ $(c \le k \le N_{\max})$ queue is employed in this paper to describe their overall charging process, where k is the maximum number of customers being served or waiting in the queue, and $N_{\max}$ is the maximum number of possible customers to be served. Similarly, it is also assumed here that their arrival and charging pattern here can be described by the Poisson distribution.

The difference between a charging station and a residential community is: in the latter scenario, the EV charging slots are generally privately owned or shared only by the residents, so the number of possible customers to be served is limited, compared to the first scenario where the possible customer number is unlimited. Such a difference leads to using different queue models to describe the charging behavior of the PHEVs.

Accordingly, the number of PHEVs being charged at the same time in an $M/M/c/k/N_{\max}$ queue follows another discrete distribution as follows:

$$p_n = \begin{cases} \left(\sum_{i=0}^{c} \binom{N_{\max}}{i} \cdot (c\rho)^i + \sum_{i=c+1}^{k} \frac{N_{\max}! \cdot (c\rho)^i}{(N_{\max}-i)! \cdot c! \cdot c^{i-c}}\right)^{-1} & n = 0 \\ (c\rho)^n \cdot \binom{N_{\max}}{n} \cdot p_0 & n = 1, 2, \ldots, c. \end{cases} \tag{10}$$

According to the above assumption, the service time T for charging a PHEV in the above two queue models follows the exponential distribution with mean T_μ, i.e.,

$$T = -T_\mu \cdot \ln(U) \tag{11}$$

where U is a uniformly distributed variate in (0, 1).

However, since it is not reasonable that a PHEV is being charged for a very short time, and the charging time also has an upper limit due to its battery capacity or service restrictions, T is truncated within a certain range $[T_{\min}, T_{\max}]$ in this paper. Therefore, T becomes

$$T = \begin{cases} T_{\min} & T \le T_{\min} \\ -T_\mu \cdot \ln(U) & T_{\min} < T < T_{\max} \\ T_{\max} & T \ge T_{\max}. \end{cases} \tag{12}$$

On the other hand, it is reported in [27] that currently there mainly exist 3 charging power levels for PHEVs, and it is reasonable that an EV charging station prefers a higher charging power level so that the service time for charging a PHEV could be reduced by supplying higher charging power. In contrast, the charging slots distributed in a residential community are limited by the distribution capacity, so their charging power level is usually limited to level 1. Therefore, in this paper, a level 3 (400V/63A) is selected for the charging station and a level 1 (230V/16A) for the residential community.

Once the charging power level is known, the charging voltage V and maximum charging current $I_{\max}$ are determined. Then the average charging current of a PHEV can be calculated by

$$I = \min\left\{\frac{D_E}{V \cdot T}, I_{\max}\right\}. \tag{13}$$

Finally, for all the n PHEVs being charged at an EV charging station or in a residential community, their total charging demand P is

$$P = \sum_{i=1}^{n} V \cdot I_i \tag{14}$$

where I_i is the charging current of the ith EV obtained from (13).

C. Different PHEV Categories

In order to consider the difference in PHEV parameters, such as k_{EV}, C_{Bat}, E_m, and M_E, PHEVs are divided into 4 classes in this paper, according to their possible k_{EV} and C_{Bat}, as shown in Table I.

Moreover, their market shares are assumed to be given by Table II.

TABLE I
C_{Bat} RANGE OF EACH EV CLASS

Class	Min C_{Bat} (kWh)	Max C_{Bat} (kWh)
Micro car	8	12
Economy car	10	14
Mid-size car	14	18
Light truck/SUV	19	23

TABLE II
MARKET SHARE OF EACH EV CLASS

Class	Percentage (100%)
Micro car	0.2
Economy car	0.3
Mid-size car	0.3
Light truck/SUV	0.2

Because the market share can be viewed as a discrete distribution, the class of a PHEV is randomly selected according to the defined market shares.

D. Random Simulation of PHEV Charging Demand

After establishing the distributions and relations of the main factors that determine the charging behavior of PHEVs, a random simulation could be applied to obtain their total charging demand samples. The procedure of this random simulation is outlined as follows:

1. Randomly generate the number of PHEVs being charged at the same time n, according to (8) or (10). Then for each one among the n PHEVs, do the following Step $2 \sim 8$:
2. Randomly select the class of an PHEV according to its market share in Table II;
3. Randomly generate PHEV parameters k_{EV} and C_{Bat} according to (2);
4. Calculate PHEV energy consumption per mile E_m according to (3);
5. Randomly generate PHEV driven miles M_d according to (4) and (5);
6. Calculate recharge energy D_E according to (6);
7. Randomly generate charging time T according to (12);
8. Calculate charging current I according to (13);
9. Accumulate total charging demand P according to (14).

The above procedure is repeatedly executed until enough samples are generated for further statistical analysis.

E. Distribution Fitting of PHEV Overall Charging Demand

In this section, two applications of the proposed methodology to model the total charging demand of PHEVs at a charging station and in a residential community are presented with specified parameters.

By referring to the experimental data in [22] and [28], the constants A_E and B_E in (3) are given in Table III.

It is assumed that M_d has a mean $\mu_{M_d} = 40$ miles and a deviation $\sigma_{M_d} = 20$ miles, in order to approximate the statistical data on EV driving distance in [28].

TABLE III
E_m PARAMETERS OF EACH EV CLASS

Class	A_E (kWh/mile)	B_E
Micro car	0.3790	0.4541
Economy car	0.4288	0.4179
Mid-size car	0.5740	0.4040
Light truck/SUV	0.8180	0.4802

TABLE IV
k_{EV} RANGE OF EACH EV CLASS

Class	Min k_{EV}	Max k_{EV}
Micro car	0.2447	0.5976
Economy car	0.2750	0.6151
Mid-size car	0.2939	0.5475
Light truck/SUV	0.3224	0.4800

TABLE V
QUEUE MODEL PARAMETERS

Parameter \ Queue Model	$M/M/c$	$M/M/c/k/N_{\max}$
T_λ (min)	10	30
T_μ (min)	60	240
c	30	100
k	–	100
$N_{\max}$	–	100
$T_{\min}$ (min)	10	120
$T_{\max}$ (min)	120	360
Voltage (V)	400	230
$I_{\max}$ (A)	63	16

Because M_E of different PHEV classes could range widely, it is selected to be μ_{M_d} for simplicity. Thus, the range of k_{EV} can be calculated from (7)

$$k_{\mathrm{EV}} = \sqrt[B_E]{\frac{C_{\mathrm{Bat}}}{(A_E \cdot M_E)}}. \tag{15}$$

By substituting Min C_{Bat}, Max C_{Bat} in Table I and μ_{M_d} into (7), the possible ranges of k_{EV} is obtained, as shown in Table IV.

Then k_{EV} and C_{Bat} are described by a bivariate normal distribution with parameters as follows:

$$\begin{aligned} \mu_{k_{\mathrm{EV}}} &= \frac{(\mathrm{Min}k_{\mathrm{EV}} + \mathrm{Max}k_{\mathrm{EV}})}{2} \\ \mu_{C_{\mathrm{Bat}}} &= \frac{(\mathrm{Min}C_{\mathrm{Bat}} + \mathrm{Max}C_{\mathrm{Bat}})}{2} \end{aligned} \tag{16}$$

$$\begin{aligned} \sigma_{k_{\mathrm{EV}}} &= \frac{(\mathrm{Max}k_{\mathrm{EV}} - \mathrm{Min}k_{\mathrm{EV}})}{4} \\ \sigma_{C_{\mathrm{Bat}}} &= \frac{(\mathrm{Max}C_{\mathrm{Bat}} - \mathrm{Min}C_{\mathrm{Bat}})}{4} \end{aligned} \tag{17}$$

$$\sum = \begin{bmatrix} \sigma^2_{k_{\mathrm{EV}}} & \rho_\Sigma \sigma_{k_{\mathrm{EV}}} \sigma_{C_{\mathrm{Bat}}} \\ \rho_\Sigma \sigma_{k_{\mathrm{EV}}} \sigma_{C_{\mathrm{Bat}}} & \sigma^2_{C_{\mathrm{Bat}}} \end{bmatrix} \tag{18}$$

where the correlation coefficient ρ_Σ is arbitrarily set to 0.8.

The queue model parameters and charging level parameters are listed in Table V, which are selected to simulate a busy period.

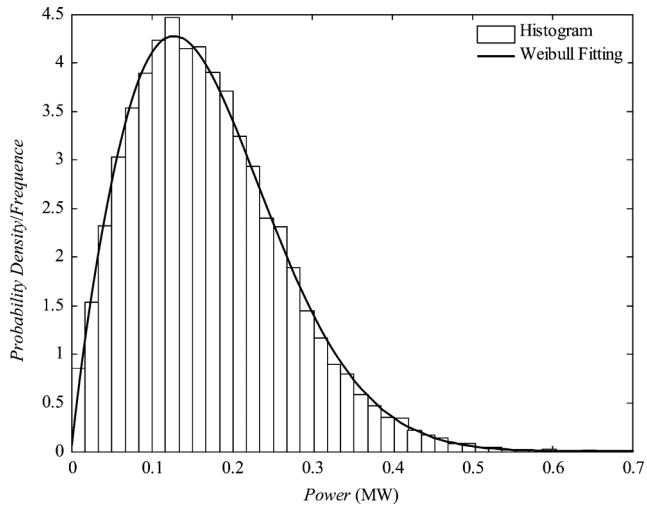

Fig. 1. Sample histogram and probability density fitting.

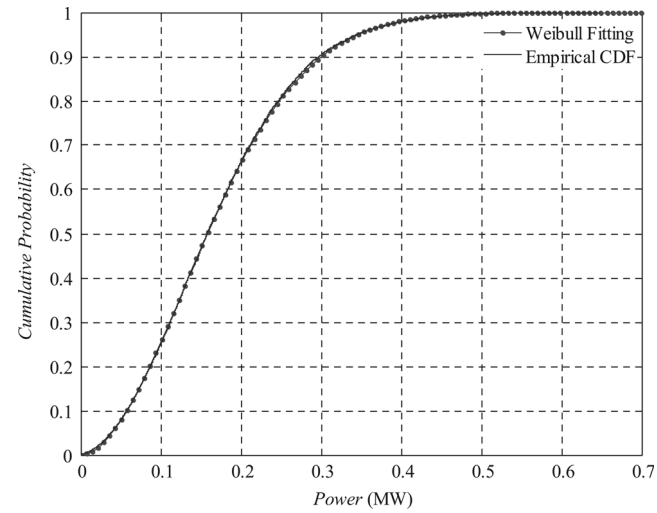

Fig. 2. K-S test for cumulative distribution fitting.

The histogram of the generated samples for the total demand of an EV charging station is shown in Fig. 1. The sample capacity is 10 000, which is adequate because the shape of this histogram changes little when the number of samples increases further.

It is hypothesized that the recharge power here follows the Weibull distribution

$$f(x|a,b) = \begin{cases} \frac{b}{a}\left(\frac{x}{a}\right)^{b-1} e^{-(x/a)^b} & x \geq 0 \\ 0 & x < 0. \end{cases} \tag{19}$$

where through maximum likelihood estimation [29], it is obtained that $a = 0.192329$ and $b = 1.87103$ with confidence degree $1 - \alpha = 95\%$.

The fitted Weibull probability density function (PDF) is also plotted in Fig. 1.

In order to validate the above hypothesis, Kolmogorov-Smirnov goodness-of-fit test (K-S test) [30] with significance level $\alpha = 5\%$ is done: K-S test statistics is 0.007413, much less than its cut-off value 0.013564, i.e., the above hypothesis is accepted, so the total charging demand of an EV charging station is identified to follow the Weibull distribution.

For comparison, the empirical cumulative distribution function (CDF) of the recharge power samples and the fitted Weibull CDF is shown in Fig. 2.

In a similar way, the charging demand of PHEVs in a residential community is identified to follow the normal distribution with the same significance level $\alpha = 5\%$, as shown in Figs. 3 and 4.

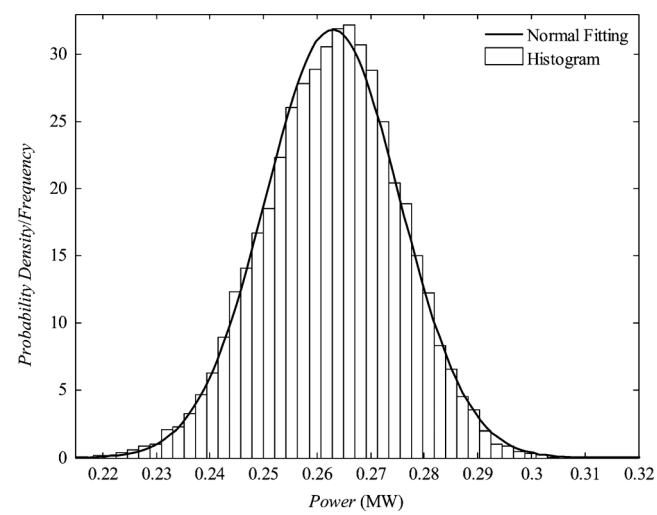

Fig. 3. Sample histogram and probability density fitting.

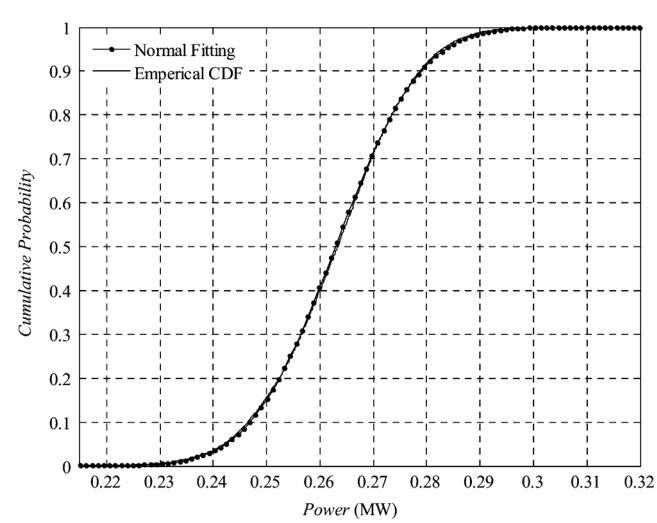

Fig. 4. K-S test for cumulative distribution fitting.

III. Case Study

In order to validate the effectiveness of the proposed modeling methodology for PPF calculations, case studies are done in an adaptation of the IEEE 30-bus test system [31], in comparison with MCS. The main modification on this regional transmission system is that two UKGDS-EHV5 urban distribution systems are connected at bus 7 and 8 respectively, replacing their original loads. The grid topology of the UKGDS-EHV5 urban distribution system is shown in Fig. 5.

As shown in Fig. 5, the PHEV charging demands are attached at the load buses in the UKGDS-EHV5 system, where a dot arrow represents the overall charging demands of PHEVs in a residential community, which are modeled as the normal distribution with the parameters shown in Table VI. On the other hand, a solid arrow represents the overall charging demand of

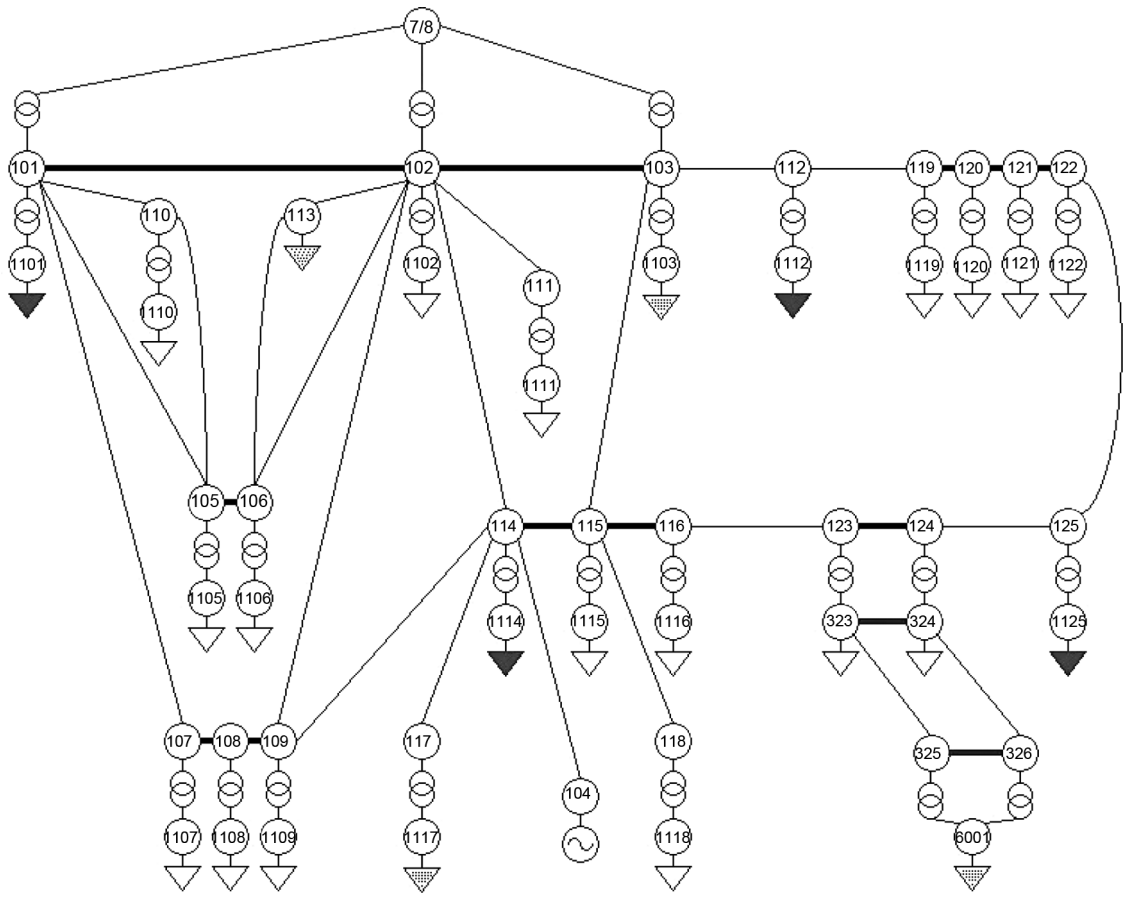

Fig. 5. UKGDS-EHV5 urban distribution system.

TABLE VI
GAUSSIAN DEMAND PARAMETERS

Bus ID	Gaussian Parameter		Power Factor
	μ_P (p.u.)	σ_P / μ_P	
113	0.20	15%	0.97014
1103	0.097689	10%	0.94551
1117	0.110227	10%	0.94551
6001	0.162325	15%	0.97918

TABLE VII
WEIBULL DEMAND PARAMETERS

Bus ID	Weibull Parameter		Mean Power (p.u.)	Power Factor
	a_P	b_P		
1101	0.192329	1.87103	0.17076	0.95
1112	0.11045	2.33015	0.097865	0.95
1114	0.192329	1.87103	0.17076	0.95
1125	0.11045	2.33015	0.097865	0.95

an EV charging station. Two sizes of charging stations are considered: the two at bus 1101 and 1114 have $c = 30$ and the other two have $c = 17$. For PPF, their charging demands are modeled as the Weibull distributions with parameters given in Table VII.

Note that these charging demands are scaled from that of a single residential community or EV charging station, in order to emphasize their size effect in the UKGDS-EHV system.

For MCS, the charging demands are directly generated from the random simulation procedure in Section II-D. The sample capacity $N_S = 10000$, which is enough for the convergence of MSC in this case.

The comprehensive algorithm proposed in [32] is employed for PPF calculations, where the mean values of voltages and branch powers are calculated by point estimate method and their higher order numerical characteristics are calculated by cumulant method.

All programs are written based on MATPOWER 4.0 [33] in MATLAB 7.11 and run on a desktop PC with a Core Duo E7200 2.53 GHz CPU, 2 GB DDR2-800 RAM. The programs take 1.68 s for PPF and 77.94 s for MCS respectively.

Table VIII includes relative errors of mean μ, variance σ^2, skewness γ_1, and kurtosis γ_2 of bus voltages and branch power flows respectively. The relative error ε_x of the numerical characteristic x is defined as

$$\varepsilon_x = \left| \frac{x_{\mathrm{PPF}} - x_{\mathrm{MCS}}}{x_{\mathrm{MCS}}} \right| \times 100\% \tag{20}$$

where x_{PPF} is the result of PPF, and x_{MCS} is the result of MCS.

As shown in Table VIII, the errors increase as the order of numerical characteristics increases, especially for the branch power flows. Nevertheless, the overall accuracy is still satisfactory. This means that it is feasible to substitute the total charging demand of substantial PHEVs with a single distribution modeled through the proposed methodology.

In order to demonstrate the approximation effect of the fitted distributions obtained from the proposed methodology, the CDFs of some bus voltage and branch power flows are plotted in Figs. 6–8.

TABLE VIII
AVERAGE AND MAXIMUM RELATIVE ERROR IN % BETWEEN PPF AND MCS

Numerical Characteristic	Avg. ε (%)	Max ε (%)
μ_V	0.211	0.401
σ^2_V	1.136	3.521
$\gamma_{1,V}$	2.769	5.786
$\gamma_{2,V}$	4.361	10.08
μ_S	0.981	2.134
σ^2_S	2.115	4.691
$\gamma_{1,S}$	6.083	8.437
$\gamma_{2,S}$	10.747	17.318

Up to 5-order Cornish-Fisher expansion [34] is employed to plot the CDF curves obtained by PPF, which is implemented with a recursive form of high performance [35], because Cornish-Fisher expansion performs better than the conventional Gram-Charlier expansion, especially when asymmetric distribution is considered in PPF [36], e.g., the Weibull distribution in this case.

As shown in the above figures, the empirical CDF curves obtained from MCS is approximated well by the results of PPFs based on the proposed methodology, and the error only increases a bit at the tail part of CDF curves. Such errors mainly result from both the PPF algorithm itself and the proposed statistical models.

In Figs. 6–8, the relative errors for the 90% quantile are 0.0060%, 0.4762% and 0.5929% respectively, and the relative errors for the 10% quantile are 0.0085%, 0.0499% and 0.3552%. The average relative errors for the 90% and 10% quantile of all bus voltage magnitudes are 0.0071% and 0.0083% respectively; in contrast, the average relative errors of all branch power flows are 2.315% and 2.984%. This average relative error of the $p\%$ quantile is defined as

$$\frac{1}{N_{\text{total}}}\sum_{i=1}^{N_{\text{total}}}\left|\frac{q^{\text{PPF}}_{p\%,i}-q^{\text{MC}}_{p\%,i}}{q^{\text{MC}}_{p\%,i}}\right|\times 100\% \tag{21}$$

where $q^{\text{PPF}}_{p\%,i}$ and $q^{\text{MC}}_{p\%,i}$ are the $p\%$ quantile obtained from PPF and MCS of the bus i or branch i, and N_{total} is the total number of buses or branches.

On the other hand, the voltage magnitude at the connection bus 7 varies only in a small range, as shown in Fig. 6. A similar phenomenon occurs at the other connection bus 8 as well. This is because the distribution system has abundant reactive power to support the bus voltages by itself and send back to the transmission system, e.g., as shown in Fig. 8. In contrast, the charging demands drawn by the distribution systems from the transmission system fluctuate in a much wider range, e.g., as shown in Figs. 7 and 8. A bootstrap analysis of the results shows that for most branches, the obtained value is with the 95% confidence limit, i.e., the dispersity of the power flows obtained from MCS is relatively high, and the results obtained from PPF also support this.

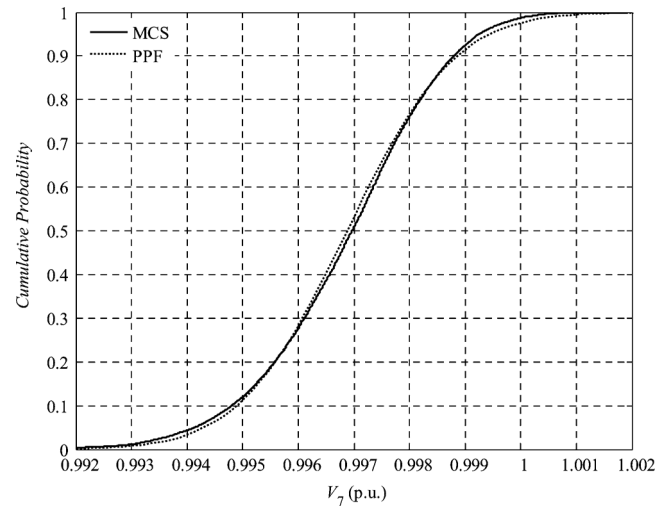

Fig. 6. Voltage magnitude CDF of bus 7.

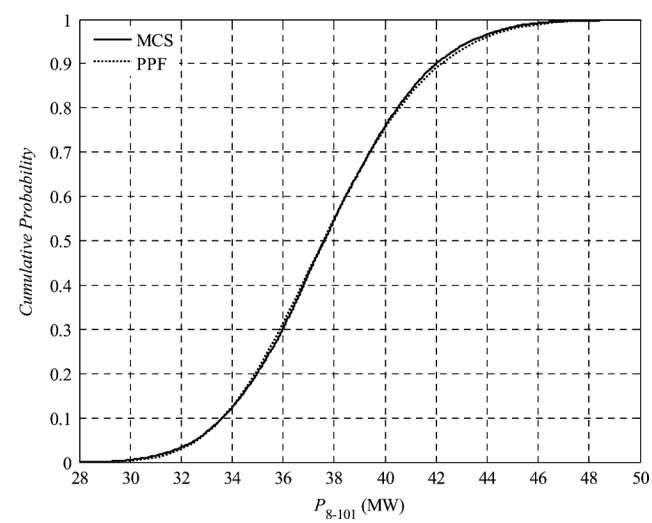

Fig. 7. Active power flow CDF of line 8 to 101.

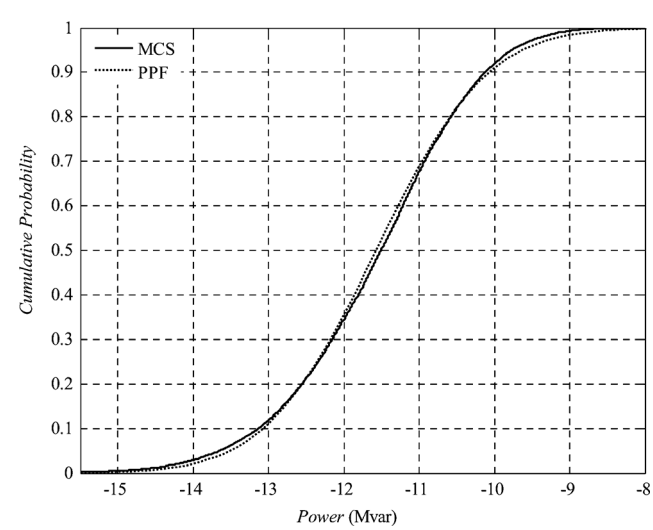

Fig. 8. Reactive power flow of line 8 to 101.

IV. CONCLUSION

This paper has proposed a methodology of modeling the overall charging demand of substantial PHEVs for PPF calculations, taking key factors that determine the charging behavior of PHEVs into consideration. The proposed methodology was further applied to modeling the overall charging demand of PHEVs at an EV charging station and in a residential community, respectively. The obtained distributions of the PHEV

overall charging demand were tested in PPF calculations through case studies, in comparison with MCS. The numerical results have shown the effectiveness of the proposed methodology. However, it should also be pointed out that the proposed methodology only considers the uncontrolled charging of PHEVs, regardless of the possible charging control strategies, such as smart charging, which should be further researched in future.

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